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Profit Allocation in Logistics Enterprise Coalitions Based on Fuzzy Cooperative Game Theory

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***Abstract*—In the context of e-commerce, establishing a stable alliance is crucial in the logistics industry. Consequently, the challenge of ensuring a fair profit allocation arises. In this study, we address the problem of profit allocation for logistics enterprise coalitions with inadequate information and propose a relevant model. To demonstrate the effectiveness of the model, a case study is provided. The results show that our method enhances the multi-party cooperation and serves as an effective tool for the fair and equitable allocation of profits.**

***Index Terms*—Fuzzy cooperative game theory, Intuitionistic fuzzy set, Logistics enterprise coalition, Profit allocation**

1. INTRODUCTION

In cooperative games, factors such as cooperation risks and personal preferences often result in partial participation. Aubin introduced the concept of cooperation ratio represented by real numbers in closed interval [1]. Butnariu defined a simplified form of Shapley function for cooperative games with fuzzy coalitions and proportional values [2]. The precise knowledge of achievable payments by each coalition is also often unavailable. Galindo and Gallardo introduced a ranking approach for fuzzy numbers and proposed real Shapley values for games with fuzzy characteristic functions [3]. Liu and Li developed two quadratic programming models to obtain the players' weighted least square contributions [4].

This paper considers a scenario where the participation levels and the profits of the alliance are both uncertain. To address this, we propose several fuzzy Shapley values.

2. METHODOLOGY

In this section, some fundamental concepts on fuzzy cooperative games are presented.

2.1 The Uncertainty of Participation Levels

The uncertainty of participation levels is often referred to as the problem of fuzzy coalitions in cooperative game theory. Intuitionistic fuzzy sets have turned out to be a more practical tool to handle the vagueness of participation levels. $\widetilde{S}(i) = \{\langle \mu_s(i), \nu_s(i)\rangle\}$ is an intuitionistic fuzzy set. $0 \leq \mu_s(i) + \nu_s(i) \leq 1,\ \mu_s(i) \in [0,1],\ \nu_s(i) \in [0,1],\ \pi_s(i) = 1 - \mu_s(i) - \nu_s(i)$. $\mu_s(i), \nu_s(i), \pi_s(i)$ represent the degree of participation,non-participation and hesitation for partner i to join the alliance $\widetilde{S}$. The intuitionistic fuzzy coalition of the coalition $S \subseteq N$ can be represented as $\widetilde{S}(i) = (\{\langle 1, \mu_s(1), \nu_s(1)\rangle\}, \{\langle 2, \mu_s(2), \nu_s(2)\rangle\}, \ldots, \{\langle n, \mu_s(n), \nu_s(n)\rangle\})$. The whole intuitionistic fuzzy sets are denoted as $IF(N)$.

Assume $[\widetilde{S}_{j_l}] = \{i \in N | \mu(i) \geq j_l\}$, $[\widetilde{S}_{k_m}] = \{i \in N | \omega(i) \geq k_m\}$, $\omega(i) = 1 - \mu(i)$, $j_l, k_m \in [0,1]$, $j_0 = 0, l = 1, 2, \ldots, x(\widetilde{s})$; $k_0 = 0, m = 1, 2, \ldots, x'(\widetilde{s})$. $[\widetilde{S}_{j_l}]$ is the set of partners whose minimum participation level is no less than j_l. $[\widetilde{S}_{k_m}]$ is the set of partners whose maximum participation level is no less than k_m. Let $X(\widetilde{S}) = \{\mu(i) | \mu(i) > 0, i \in N\}$, $X'(\widetilde{S}) = \{\omega(i) | \omega(i) > 0, i \in N\}$, $x(\widetilde{s})$, $x'(\widetilde{s})$ correspond to the number of elements in $X(\widetilde{S})$, $X'(\widetilde{S})$. By applying the Choquet integral, we can obtain the characteristic function of the intuitionistic fuzzy coalition cooperative game

$$\overline{v}(\widetilde{S}) = [\overline{v}^-(\widetilde{S}), \overline{v}^+(\widetilde{S})] \tag{1}$$

where $v^-(\widetilde{S}) = \sum_{l=1}^{x(\widetilde{s})} v(\widetilde{S}_{j_l})(j_l - j_{l-1})$, $v^+(\widetilde{S}) = \sum_{m=1}^{x'(\widetilde{s})} v(\widetilde{S}_{k_m})(k_m - k_{m-1})$. The pair $\langle IF(N), \overline{v}\rangle$ is a typical form of intuitionistic fuzzy coalition cooperative games.

Given an intuitionistic fuzzy coalition cooperative game, its Shapley value is

$$\overline{\varphi_i} = [\varphi_i^-(\overline{v}), \varphi_i^+(\overline{v})] \tag{2}$$

where $\varphi_i^-(\overline{v}) = \sum_{i=1}^{x(s)} \varphi_i[v(\widetilde{S}_{j_l})](j_l - j_{l-1})$, $\varphi_i^+(\overline{v}) = \sum_{i=1}^{x'(s)} \varphi_i[v(\widetilde{S}_{k_m})](k_m - k_{m-1})$. $\varphi_i[v(\widetilde{S}_{j_l})]$ is the Shapley value of all the participants whose participation level satisfies $\mu(i) \geq j_l$. $\varphi_i[v(\widetilde{S}_{k_m})]$ is the Shapley value of all the participants whose participation level satisfies $\omega(i) \geq k_m$.

2.2 The Uncertainty of Payoffs

The expected profits correspond to each participant's characteristic function. We use the triangular fuzzy numbers to represent coalitions' values as the method depicts the uncertainty of profits in a more applicable way.

A β-cut set of the triangular number $\widetilde{a} = (a^l, a^m, a^r)$ is defined as $\widetilde{a}(\beta) = \{x | \mu_{\widetilde{a}}(x) \geq \beta\}$, where $\beta \in [0,1]$. Thus, a β-cut set of the triangular number $\widetilde{a} = (a^l, a^m, a^r)$ is an interval number, denoted as $\widetilde{a}(\beta) = [a_L(\beta), a_R(\beta)]$ where $a_L(\beta) = (1-\beta)a^l + \beta a^m$ and $a_R(\beta) = (1-\beta)a^r + \beta a^m$.

We define an n-person triangular fuzzy cooperative game through an ordered pair $\langle N, \widetilde{v}\rangle$. $N = \{1, 2, \ldots, n\}$ is the set of players and $\widetilde{v}$ is the triangular fuzzy characteristic function of players' coalitions. In general, for any coalition $S(S \subseteq$

N), $\widetilde{v}(S)$ is defined by the triangular fuzzy number $\widetilde{v}(S) = (v^l(S), v^m(S), v^r(S))$, where $v^l(S) \leq v^m(S) \leq v^r(S)$ and $\widetilde{v}(\varnothing) = (0, 0, 0)$.

In order to achieve a fair and balanced profit allocation that maximizes the satisfaction of each participant, we introduce the concept of an excess vector. It measures the satisfaction of partners in the allocation. Given any payoff $x \in R^n$ and participants $i (i \in N)$, an excess vector is defined as $e(i, x) = v(i) - x(i)$ And the distance formula of an α-cut set of the triangular number is given as

$$D(\widetilde{a}(\alpha), \widetilde{b}(\alpha)) = (a_L(\alpha) - b_L(\alpha))^2 + (a_R(\alpha) - b_R(\alpha))^2 \quad (3)$$

Now we can construct a mathematical model to minimize the dissatisfaction of the participants.

$$min\{\sum_{i \in N} [v_L(i)(\alpha) - x_L(i)(\alpha)]^2 + \sum_{i \in N} [v_R(i)(\alpha) - x_R(i)(\alpha)]^2\}$$

$$s.t. \begin{cases} \sum_{i \in N} x_L(i)(\alpha) = v_L(N)(\alpha) \\ \sum_{i \in N} x_R(i)(\alpha) = v_R(N)(\alpha) \end{cases} \quad (4)$$

2.3 Uncertainty of Dual Fuzzy Elements

In this section, we combine the fuzziness of both participation levels and payoffs. Let $N = \{1, 2, \ldots, n\}$ be the set of participants, all the sets of intuitionistic fuzzy sets on N be denoted as $G_{if}(N)$, $\widetilde{v}$ be denoted as a triangular fuzzy number on $G_{if}(N$, which satisfies $\widetilde{v} : G_{if}(N) \to \widetilde{R}$, $\widetilde{v}(\varnothing) = 0$. We define $\langle N, \widetilde{v} \rangle$ as an intuitionistic fuzzy coalition cooperative game whose payoff is a triangular number.

Assume $\widetilde{S} \in G_{if}(N)$. Let $X^-(\widetilde{S}) = \{\mu(i) | \mu(i) > 0, i \in N\}$, $X^+(\widetilde{S}) = \{1 - \nu(i) | 1 - \nu(i) > 0, i \in N\}$, $x^-(\widetilde{s})$, $x^+(\widetilde{s})$ correspond to the amount of elements in $X^-(\widetilde{S})$, $X^+(\widetilde{S})$ respectively. Arrange the elements in $X^-(\widetilde{S})$ and $X^+\widetilde{S}$ in monotonically increasing order as $h_1^- < h_2^- < \cdots < h_{x^-(\widetilde{S})}$, $h_1^+ < h_2^+ < \cdots < h_{x^+(\widetilde{S})}$. Similarly, we can obtain the formula of the Choquet integral of an intuitionistic fuzzy game

$$\gamma^-(\widetilde{S}) = \sum_{i=1}^{x^-(\widetilde{S})} v([\widetilde{S}]_{h_l^-})(h_l^- - h_{l-1}^-) \quad (5)$$

$$\gamma^+(\widetilde{S}) = \sum_{i=1}^{x^+(\widetilde{S})} v([\widetilde{S}]_{h_l^+})(h_l^+ - h_{l-1}^+) \quad (6)$$

Given a confidence level α, v_α be the aggregation form of v at α, we can obtain the Shapley function of $\langle N, v_\alpha \rangle$ as

$$\phi_i(v_\alpha) = \sum_{\substack{i \in \widetilde{S} \\ \widetilde{S} \in G_{if}(N)}} \frac{(|\widetilde{S} - 1|)!(n - \widetilde{S})!}{n!} (v_\alpha(\widetilde{S}) - v_\alpha(\widetilde{S} - \langle i, \mu(i), \nu(i) \rangle)) \quad (7)$$

3. Case study

One example is set at a county with several logistics enterprises. In rural areas, the delivery of goods is currently affected by factors such as long distances and complex road conditions. Consequently, most logistics companies have not yet established comprehensive coalitions. To gain a higher profit and save more costs, the logistics enterprises adopt the collaborative delivery model. We select three typical logistics companies. Their profits when they operate individually are given in Table 1.

TABLE I
THE INDIVIDUAL PROFITS OF THREE LOGISTICS COMPANIES

Enterprises	Incomes	Costs	Profits
A	[751,2263,3771]	3565	[-2814,-1302,206]
B	[1513,3023,4537]	3565	[-2052,-542,972]
C	[5307,6773,8317]	3565	[1742,3208,4752]
Sum	[7571,12059,16625]	10695	[-3124,1364,5930]

With the coalition formed, the companies can integrate their transportation resource. We apply the method in Section 2.1, 2.2 and 2.3 to calculate the profits allocated to logistics enterprise A, B and C. The results are given in Table 2.

TABLE II
PROFITS ALLOCATED UNDER DIFFERENT STRATEGIES

Strategy	Enterprise A	Enterprise B	Enterprise C
Fuzzy Coaltion	[-804,-412,442]	[-657,12,1573]	[3015,4906,7305]
Fuzzy Payoff	[-633,-353,457]	[-615,57,1615]	[2802,4802,7248]
Dual Elements	[-589,-291,503]	[-561,1644,]	[2704,4695,7173]

The results demonstrate that when logistics enterprises establish coalitions, they are able to achieve higher profits compared to operating in isolation. The overall profit of the local logistics industry can also get enhanced.

4. Conclusions

In this paper, by combining traditional cooperative game theory with Choquet integral, intuitionistic fuzzy sets and triangular fuzzy numbers, we propose several solutions for fuzzy cooperative games to address more practical problems. The case study suggests that the profits of all the participants shall get increased through the proposed approach.

We will make further research from the following two aspects: (1) Consider more details about the companies and develop a more comprehensive model. (2) When calculating the fuzzy Shapley values, take influencing factors such as risks and innovation into account.

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