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High Frequency Data-Driven Dynamic Portfolio Optimization for Cryptocurrencies

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***Abstract*—Recently there has been a growing interest in constructing portfolios with stocks and cryptocurrencies. As cryptocurrency prices increase over the years, there is a growing interest in investing in cryptocurrencies, along with diversifying portfolios by adding multiple cryptocurrencies to the existing portfolios. Even though investing in cryptocurrency leads to high returns, it also leads to high risk due to the high uncertainty of cryptocurrency price changes. Thus, more robust risk measures have been introduced to capture market risk and avoid investment loss, along with different types of portfolios to mitigate risks. Many portfolio techniques assume asset returns are normally distributed with constant variance. However, these assumptions are violated in many cases. Unlike the existing work, this study investigates the recently proposed data-driven exponentially weighted moving average (DDEWMA) covariance model to estimate the variance-covariance matrix for high frequency (hourly data) cryptocurrency returns in Markowitz portfolio optimization. The experimental results show that for high-frequency data, the DDEWMA approach outperforms the existing portfolio optimization model that uses the empirical variance-covariance matrix. Improvements have been identified in terms of the Sharpe ratio as well as risks (volatility, mean absolute deviation (MAD), Value-at-Risk (VaR), and Expected shortfall (ES)).**

***Index Terms*—Cryptocurrencies, Portfolio Optimization, Sharpe ratio, Volatility**

I. Introduction

Numerous data sources follow a sequential pattern and necessitate distinctive handling while constructing predictive models. The task of prediction becomes challenging, especially when dealing with financial time series data like trading volumes, stock and bond prices, and exchange rates. Despite the complexities, certain indices can still be reasonably forecasted with a degree of accuracy [1].

There is a growing interest in investing in different types of stocks as a risk diversification strategy. Following the financial crisis in 2008, the governing authorities have imposed on financial institutions to use more robust risk measures incorporating market risk to reduce the potential loss of investments [2] in the future. Incorporating risk metrics like volatility, value-at-risk (VaR), and Expected Shortfall (ES) into the decision-making process serves as a means to mitigate the potential risks of financial failures. Numerous studies have explored how these diverse risk measures can facilitate efficient capital allocation and enhance risk management practices.

In 2008, the digital currency was introduced with the introduction of Bitcoin [3]. A cryptocurrency is a form of digital currency whose value is solely derived from the trust placed in it. Digital currencies are transforming the conventional financial system at a rapid pace, primarily because they eliminate the need for financial intermediaries, eliminate transactional delays, and reduce paperwork, among other benefits. As Bitcoin prices increase over the years, there is an uprise in investing in cryptocurrencies along with diversifying portfolios by adding multiple cryptocurrencies to the existing portfolios [4] and [5].

In 1952, Markowitz introduced a framework for computing the ideal asset allocation within an investment portfolio, pioneering what is now known as modern portfolio theory [6]. The initial model finds the optimal weights (fund allocation of initial investment) for a given level of risk, and different versions of the model have been studied since then with complicated modifications. Recently Awoye [7] studied a machine learning approach with the Markowitz model, and the idea of graphical Lasso was investigated in minimizing portfolio risk by Millington and Niranjan [8]. Furthermore, researchers have explored various methods in portfolio optimization, including the Naïve portfolio, Mean-variance portfolio, and Sharpe ratio (SR). These investigations have revealed that a diversified portfolio, inclusive of cryptocurrencies, tends to yield superior returns when compared to one that excludes them [9]. However, many of the approaches/studies assume normality and are required to estimate/calculate the inverse covariance matrix of asset price return. Thavaneswaran et al. [10] demonstrated that the assumption of normality is not applicable to the chosen stocks, and the established portfolio risk metrics are influenced by the greater skewness and kurtosis in the portfolio returns. Hence, they proposed new risk measures for portfolio optimization that incorporate high skewness and kurtosis, and recently, Bowala and Singh [11] have investigated the performance of new risk measures for high-frequency cryptocurrency data.

Zhu et al. [12] introduce a portfolio optimization method that relies on a data-driven exponentially weighted moving average (DDEWMA) model for assessing volatility. The current study extends the work of [12] by proposing a dynamic portfolio optimization with DD volatility for cryptocurrencies. Regular stock markets generally operate five days per week with eight-hour shifts, and at the end of the day, a daily adjusted closing price is reported for each stock. Therefore, most studies consider daily data in the analysis. However, cryptocurrency transactions happen twenty-four hours a day and seven days per week. Given its nature, it is important to investigate how a portfolio can be affected by this fast-moving market. Thus, this study considers high-frequency data (hourly adjusted closing prices) for cryptocurrencies in the analysis.

The rest of the paper comprises two sections followed by the conclusions. Section 2 delves into data-driven variance-covariance matrices and introduces a performance evaluation metric for the new approach. Section 3 elaborates an overview of the numerical experiments conducted for a portfolio that includes six distinct cryptocurrencies.

II. Methodology

A. DDEWMA Model for Volatility

It has been noted that the volatility of asset returns fluctuates over time [13]. Thus, models that can incorporate time-varying volatility may provide better forecasts of risks. In 2020, Thavaneswaran et al. [14] introduced a novel DDEWMA model for volatility. The model can be extended to portfolio returns (R_t) by replacing asset log returns with portfolio returns. The model is written as:

$$\sigma_{t+1} = \frac{\alpha}{\rho}|R_t - \bar{R}| + (1-\alpha)\sigma_t \quad 0 < \alpha < 1$$

where σ_t is volatility at time t and α is the smoothing constant obtained by minimizing the one step ahead forecast error sum of squares. ρ is the sign-correlation of the return,

$$\rho = Corr(R - \bar{R}, sign(R - \bar{R})).$$

If the asset returns are believed to be Student-t distributed with ν degrees of freedom (DF), ν can be estimated by solving the equation 1 (see [14] for more details).

$$2\sqrt{\nu - 2} = \rho(\nu - 1)Beta\left(\frac{\nu}{2}, \frac{1}{2}\right) \tag{1}$$

Let P_t be the adjusted closing price at time t. Then, simple return at time t (R_t) can be calculated by $R_t = (P_t - P_{t-1})/P_{t-1}$. The weighted average of asset returns ($R_{i,t}$) is known as portfolio return ($R_{p,t}$). When new observation comes, the portfolio needs to be rebalanced in high-frequency trading. This need for frequent updates on smoothed values can be handled efficiently by DDEWMA.

The training period window for the study is $[1, T_1]$. If the last data point is collected at time T_2, the test period is denoted by $[T_1 + 1, T_2]$. After establishing the optimal smoothing constant through the training data, it will then be applied to predict volatility during the testing period. The DDEWMA algorithm for computing a one-step-ahead forecast of volatility and residuals proposed in [12] is summarized in algorithm 1.

Algorithm 1 DDEWMA volatility forecasts of returns

Require: Data: Adjusted closing asset prices $P_t, t = 0, \ldots, k, \ldots, T_1, \ldots T_2$.

$R_{i,t} \leftarrow \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}, t = 1, \ldots, T_1, \ldots T_2$

$\hat{\rho} \leftarrow \text{Corr}(R - \bar{R}, \text{sign}(R - \bar{R}))$

$Z_t \leftarrow |R_t - \bar{R}|/\hat{\rho}$

$S_0 \leftarrow \frac{\sum_{t=1}^{k} Z_t}{k}$

$\alpha \leftarrow (0, 1)$

$S_t \leftarrow \alpha Z_t + (1-\alpha)S_{t-1}, t = 1, \ldots, T_1$

$\alpha_{opt} \leftarrow \min_\alpha \sum_{t=k+1}^{T_1} (Z_t - S_{t-1})^2$

for $t \leftarrow 1, T_2$ **do**

  $S_t \leftarrow \alpha_{opt} Z_t + (1 - \alpha_{opt})S_{t-1}$

  $res_t \leftarrow \frac{R_t - \bar{R}}{S_{t-1}}$

return $S_t, t = T_1, \ldots T_2$

return $res_t, t = 1, 2, \ldots T_2$

B. Optimal Portfolio Weights

As assets with high risks yield high returns in general, maximizing return leads to high risk and lower risk resulting in low return in the simplest setup in portfolio optimization. It is important to have a measure that captures both return and risk, and the Sharpe ratio (SR) is the most commonly used and popular measure that captures both risks and returns. The Portfolio Sharpe ratio (equation 2) is the risk-adjusted portfolio return, and it enabled us to compare the performance of different portfolios considering both risk and return.

$$SR = \frac{E[R_{p,t}] - \frac{r_f}{N}}{\sqrt{Var(R_{p,t})}} = \frac{\mu_p - \frac{r_f}{N}}{\sigma_p} \tag{2}$$

In equation 2 r_f is the annual risk-free rate, N is the number of tradings in one year, $\mu_p = \omega^T \mu$, and $\sigma_p = \sqrt{\omega^T \Sigma \omega}$, where ω, Σ, and μ are the vector of portfolio weights, the covariance matrix, and the mean vector of the asset return, respectively. Then, the annualized Sharpe ratio can be calculated by $\sqrt{N} * SR$.

Finding the tangency portfolio, known as the optimal risky portfolio, maximizes the Sharpe ratio given in equation 2. The constraint for this optimization problem is that the sum of portfolio weights (fund allocation from initial investment) is equal to one. Thus, the entire initial investment will be invested in the given portfolio, and $\omega_i * 100\%$ gives the percentage of funds allocated from the initial investment in asset i. The optimal weights for the tangency portfolio are given by

$$w_{Optimal} = \frac{\Sigma^{-1}(\mu - r_f)}{\mathbf{1}^T \Sigma^{-1}(\mu - r_f)}. \tag{3}$$

C. Data-Driven and Empirical Covariance Matrices

The correlation matrix is generated based on the covariance matrix of the standardized residuals. An element in the data-driven covariance matrix can be written as

$$\Sigma^{dd}[t-1]_{i,j} \leftarrow cor^{dd}[t-1]_{i,j}\hat{\sigma}_{i,t}\hat{\sigma}_{j,t}.$$

Following the equation 3, obtaining optimal weights for the portfolio using a data-driven covariance matrix is shown in the algorithm 2. Once the optimal weights are determined, the realized return for the portfolio using the testing sample is computed by

$$R_{p,t} = \sum w_{i,t-1}R_{i,t}.$$

Algorithm 2 Portfolio Weights from DDEWMA Covariance Matrix

Require: Data: Adjusted Closing Price of assets $P_{i,t}, t = 0, \ldots, k, \ldots T_1, \ldots T_2, i = 1, \ldots, n$

1: $R_{i,t} \leftarrow \frac{P_{i,t}-P_{i,t-1}}{P_{i,t-1}}, t = 1, \ldots, T_2$

2: $\hat{\sigma}_{i,t}$ is the S_{t-1} for R_i in Algorithm 1, $t = T_1 + 1, \ldots T_2$

3: $\Sigma^{dd}[t-1]_{i,i} \leftarrow \hat{\sigma_{i,t}}^2$

4: $\Sigma^{dd}[t-1]_{i,j} \leftarrow cor^{dd}[t-1]_{i,j}\hat{\sigma_{i,t}}\hat{\sigma_{j,t}}$

5: Check the matrix is positive definite

6: $Z \leftarrow \Sigma^{-1}(\mu - R_f)$

7: $w^{dd}_{i,t-1} \leftarrow \frac{Z_i}{\sum Z}$

8: **return** $w^{dd}_{i,t-1}$

In this study, apart from the Sharpe ratio, we predict various other risk metrics, including mean absolute deviation (MAD), Value-at-Risk (VaR), and Expected Shortfall (ES). The formulas for calculating the Sharpe ratio, MAD, VaR, and ES using a one-step-ahead volatility forecast ($\hat{\sigma}_{T+1}$) are provided in equations 4, 5, 6, and 7, respectively.

$$\begin{aligned} SR_{T+1} &= \frac{E[R_{T+1}|R_T, R_{T-1}, \ldots R_1] - \frac{R_f}{N}}{\sigma(R_{T+1}|R_T, R_{T-1}, \ldots R_1)}, \\ &= \frac{\hat{\mu}_p - \frac{R_f}{N}}{\hat{\sigma}_{T+1}}, \end{aligned} \tag{4}$$

$$MAD_{T+1} = \hat{\sigma}_{T+1}\hat{\rho}, \tag{5}$$

$$VaR_{T+1} = (-1000)(\hat{\sigma}_{T+1}t_\nu^{-1}(p)\sqrt{\frac{\nu-2}{\nu}}), \tag{6}$$

$$\begin{aligned} ES_{T+1} = 1000 * \hat{\sigma}_{T+1} * \\ \left(\frac{f_\nu(t_\nu^{-1}(p))}{p}\right)\left(\frac{\nu + (t_\nu^{-1}(p))^2}{\nu - 1}\right)\sqrt{\frac{\nu-2}{\nu}}, \end{aligned} \tag{7}$$

where $\hat{\mu}_p$ is the expected portfolio return and f_ν is the probability density function of a student-t distribution with ν DF.

D. Utility of Portfolio Optimization

Zhu et al. [12] proposed a new metric to quantify the performance increase of the data-driven approach against the empirical approach in terms of the Sharpe ratio (equation 8). The newly introduced measure quantifies the percentage increase in utility achieved by transitioning from the empirical variance-covariance matrix to the data-driven exponentially weighted moving average variance-covariance matrix.

$$\text{Utility Increase } \% = \frac{SR^2_{DD} - SR^2_{EMP}}{SR^2_{EMP}} \tag{8}$$

III. Experimental Results

The study considers hourly adjusted closing price data from six cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNC), Ripple (XRP), Dogecoin (DOGE), and Cardano (ADA). Hourly adjusted closing prices (USD) are downloaded from Yahoo! Finance (http://www.finance.yahoo.com) from November 2020 to May 2021. The selection of cryptocurrencies is based on their market cap (current Price x circulating supply) according to Coinmarketcap (http://www.coinmarketcap.com). Note that in Coinmarketcap, Tether has the third-highest market cap, and USD Coin has the fifth-highest market cap. However, they have not been included in this study as Tether and USD Coin are stablecoins, and they would be less attractive to be included in a portfolio.

Table I provides summary statistics for all the assets during the study period. DOGE has the highest mean return, and BTC has the lowest mean return among the selected cryptocurrencies. The Skewness of mean returns indicates that data are not significantly skewed except for DOGE. Nonetheless, for all the cryptocurrencies, kurtosis is significantly high, indicating heavy tail distributions for mean returns. The table also provides the auto-correlation function values for log returns. As these values deviate from zero, it suggests that the series exhibits significant autocorrelation, signifying the presence of volatility clustering. Based on the estimated sign correlation value ($\hat{\rho}$), the corresponding DF (ν) for student-t distribution is determined. For all the assets, simple returns indicate heavy tails with lower DF.

TABLE I
Summary Statistics of Hourly Returns

Asset	Mean	SD	Skewness	Kurtosis	ACF	$\hat{\rho}$	ν
BTC	0.0004	0.0108	0.3685	10.4182	0.0361	0.6913	3.66
ETH	0.0005	0.0126	-0.1244	7.3092	0.0288	0.6926	3.69
BNC	0.0011	0.0172	0.6385	12.4631	-0.0217	0.6467	3.09
XRP	0.0006	0.0192	0.1596	24.8716	-0.0294	0.5899	2.70
DOGE	0.0017	0.0347	3.5342	49.1230	0.0077	0.4759	2.33
ADA	0.0010	0.0179	0.4961	8.4593	-0.0650	0.6957	3.75

In this study, one month of adjusted closing prices of assets (24 hours per day * 30 days per month = 720 data points per month) are used as the training data, and portfolio forecasts are obtained using the data-driven and empirical approaches for one day (24 hours/24 data points). More importantly, we consider a dynamic setup for obtaining the portfolio forecasts, and it enables us to avoid the seasonality and cyclic effect of cryptocurrency price data. In the following subsections, SR rolling forecasts and rolling risk forecasts of portfolios are summarized using 2021 January, February, and March monthly data. Once the models are trained, one day of hourly forecasts of SR, MAD, VaR, and ES are obtained with both empirical and data-driven variance-covariance matrices. In this study, the risk-free rate is represented by the average treasury bill rate (T-bill rate). Specifically, for each month, we utilize the mean treasury bill rate of that respective month as the risk-free rate. The treasury bill rates are obtained from Bloomberg (https://www.bloomberg.com/canada).

A. *Forecasts of SR and risks using January 2021*

This subsection summarizes the forecasts using January 2021 data for both data-driven and empirical approaches. Table II summarizes means and standard deviations of SR and risks. Compared to the empirical approach, the mean SR using the data-driven variance-covariance matrix is higher with a lower standard deviation. Moreover, both means and standard deviations of volatility, MAD, VaR, and ES forecasts are higher using the empirical covariance matrix.

TABLE II
SUMMARY OF FORECASTS USING JANUARY 2021 DATA

		$\hat{\sigma}$	SR	MAD	VaR	ES
EMP	mean	0.0120	0.0960	0.0088	30.4175	42.3860
	SD	0.0020	0.0135	0.0014	5.3154	7.3874
DD	mean	0.0068	0.1430	0.0051	16.3438	22.0146
	SD	0.0005	0.0112	0.0004	1.2623	1.6178

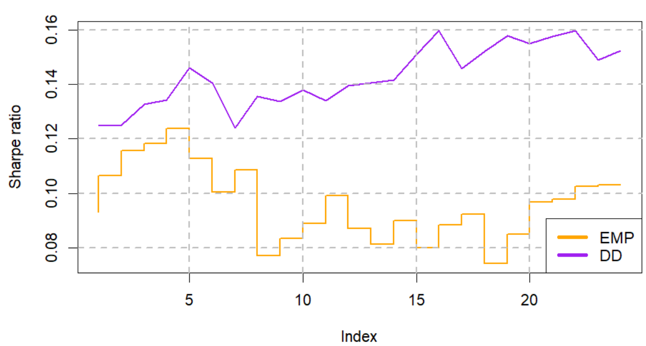

Fig. 1. SR Rolling Forecasts of Portfolios (January 2021)

Figures 1 visualizes the rolling forecasts of SR and risk for January 2021 data. SR is higher with the data-driven variance-covariance matrix, and risks are lower for each hour. This indicates that using the empirical variance-covariance matrix leads to portfolios with high risks and lower SR compared to the data-driven approach. A similar trend was observed for other risk measures: simple volatility, MAD, VaR, and ES.

Figure 2 further summarizes and visualizes the risks of the portfolio using comparison boxplots for the data-driven and empirical approaches for January 2021 data. Observe that risks using empirical covariance matrix are always high with high variability. Though it is visible from the figures that portfolios

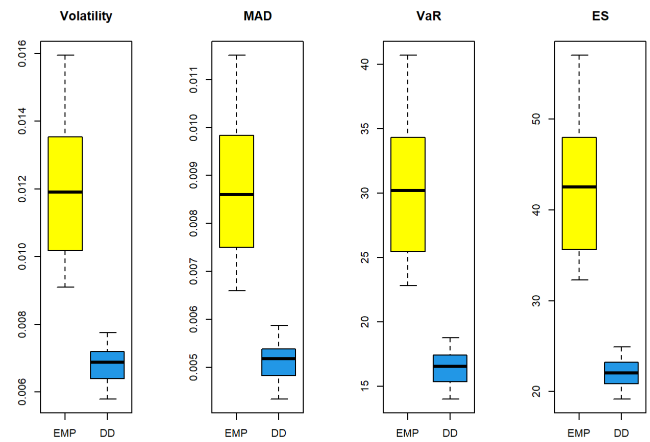

Fig. 2. Comparison Boxplots of Risks (January 2021)

constructed with data-driven covariance matrix outperform, it is also important to quantify the improvement for further clarification. The calculated increase in utility when transitioning from the empirical covariance matrix to the DDEWMA variance-covariance matrix is approximately 138.80% for data from January 2021. It is a significant improvement in SR.

B. *Forecasts of SR and risks Using February 2021 Data*

The SR and risk forecasts using data-driven and empirical variance-covariance matrices with February 2021 data are summarized in this subsection. Means and standard deviations of the forecasts are provided in Table III. Similar to January data, the SR forecast obtained with February data using the data-driven variance-covariance matrix is larger than that with the empirical variance-covariance matrix. Moreover, volatility, MAD, VaR, and ES mean forecasts are higher with the empirical variance-covariance matrix indicating the new data-driven approach is superior considering February 2021 data as well.

TABLE III
SUMMARY OF FORECASTS USING FEBRUARY 2021 DATA

		$\hat{\sigma}$	SR	MAD	VaR	ES
EMP	mean	0.0116	0.1314	0.0086	28.6721	39.4283
	SD	0.0021	0.0226	0.0016	5.4796	7.2324
DD	mean	0.0087	0.1725	0.0063	21.3815	30.7767
	SD	0.0020	0.0325	0.0015	5.2433	6.9800

Forecasts of SR obtained for the testing period (one day/twenty-four hours) using February data are visualized in Figure 3. The SR rolling forecasts with data-driven covariance matrix are higher than the SR rolling forecasts with empirical covariance matrix, except for the eleventh hour. However, it is observed that volatility, MAD, VaR, and ES forecasts using data-driven covariance matrices are consistently lower compared to the empirical approach. It is also important to point out that the gap between the forecasts using data-driven and empirical covariance matrices is smaller compared to forecast gaps considering January data. This can be seen in the comparison boxplots given in Figure 4. Nonetheless, median volatility, MAD, VaR, and ES forecasts obtained using the data-driven covariance matrix are lower compared to the forecasts obtained with the empirical covariance matrix.

Fig. 3. SR Rolling Forecasts of Portfolios (February 2021)

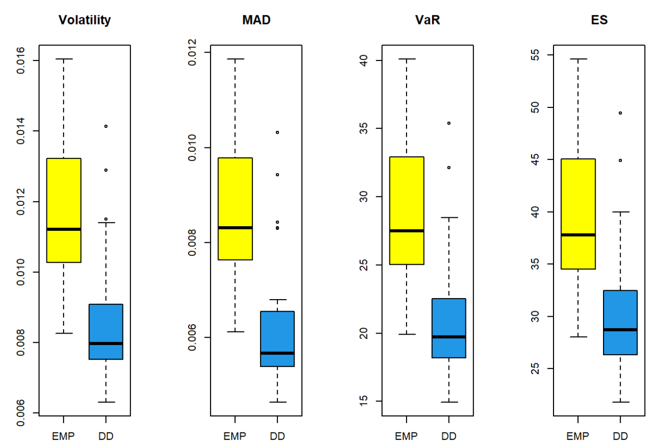

Fig. 4. Comparison Boxplots of Risks (February 2021)

Since the performances of the two approaches are close at certain testing hours, it is important to see the improvements in SR using the data-driven variance-covariance matrix over the empirical variance-covariance matrix. The estimated average change of the percentage increase is 76.86%.

C. *Forecasts of SR and risks for March 2021*

The SR and forecasts of risks using March data are also obtained, and results are summarized in Table IV. The mean SR forecast using the data-driven variance-covariance matrix is larger than the mean SR forecast obtained with the empirical variance-covariance matrix. Mean risk forecasts are still lower with the data-driven covariance matrix, and standard deviations of the forecasts are also lower compared to the empirical approach. Thus, this indicates that the data-driven approach outperforms the empirical approach using March 2021 data in terms of SR and risks.

TABLE IV
SUMMARY OF FORECASTS USING MARCH 2021 DATA

		$\hat{\sigma}$	SR	MAD	VaR	ES
EMP	mean	0.0040	0.0845	0.0031	9.7942	12.5759
	SD	0.0005	0.0060	0.0004	1.1315	1.4449
DD	mean	0.0036	0.0922	0.0027	8.8411	11.6315
	SD	0.0003	0.0142	0.0002	0.7745	1.0986

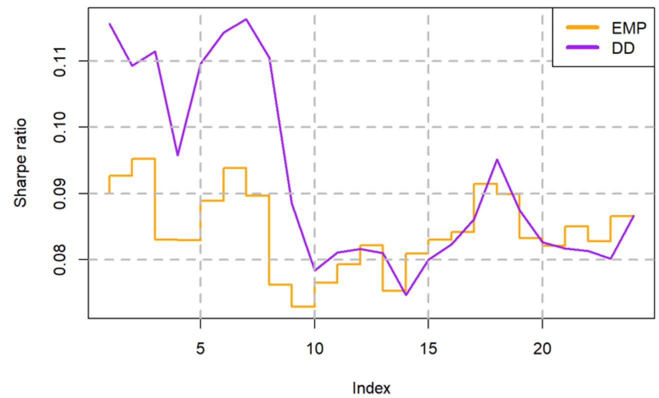

Fig. 5. SR Rolling Forecasts of Portfolios (March 2021)

We observed that for some cryptocurrencies adjusted closing prices decreased towards the end of the study period. This will bring a negative effect on portfolio returns, ultimately leading to a lower Sharpe ratio when the risk is fixed. Also, we observed sudden price increases and decreases during March 2021 for some cryptocurrencies. Therefore, during this volatile period (March), we expect somewhat different results in terms of SR and risk forecasts compared to the forecasts obtained with January and February data. The SR forecasts for the testing period (one day/twenty-four hours) using March 2021 data are given in Figure 5. During the testing period, SR forecasts using the data-driven variance-covariance matrix are higher or closer to the SR forecasts obtained with the empirical variance-covariance matrix. Similar observations can also be made for the risk forecasts, and in most instances, risks using the data-driven covariance matrix are lower. This closed performance of the empirical approach and data-driven approach can be seen with comparison boxplots given in Figure 6 for risks. In contrast to the January and February comparison boxplots for risks, boxplots for the two approaches overlap for March data. However, boxplots indicate that risk forecasts obtained with the data-driven variance-covariance matrix have lower median and lower variability compared to the risk forecasts obtained with the empirical variance-covariance matrix. This ambiguity of the performances of the two approaches can be addressed by measuring the gain in utility when switching to the data-driven approach. Using the equation 8, the percentage gain has been calculated, and for March 2021 data, 19.54% gain in utility can be obtained when switching from the empirical variance-covariance matrix to the DDEWMA variance-covariance matrix.

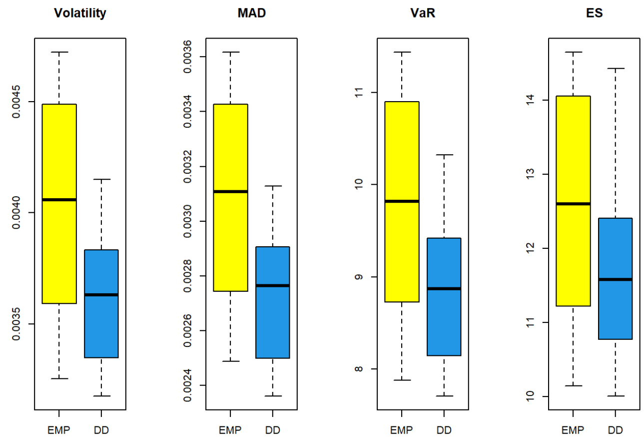

Fig. 6. Comparison Boxplots of Risks (March 2021)

IV. Conclusion

This paper extends the portfolio optimization based empirical covariance matrix to dynamic portfolio optimization with high frequency (hourly) cryptocurrency data. The driving idea, unlike the existing work, is studying dynamic portfolio optimization without assuming normality for portfolio returns. Our experimental findings demonstrate that when constructing portfolios with cryptocurrencies, the new approach surpasses the commonly used portfolio optimization method based on the empirical variance-covariance matrix. This superiority is evident through higher Sharpe ratios and reduced risks.

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