2023 IEEE Symposium Series on Computational Intelligence (SSCI)
Mexico City, Mexico. December 5-8, 2023

Analyzing Different Protocols of Information Granularity Distribution to Improve Consistency of Fuzzy Preference Relations in Decision-Making

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***Abstract*—A fundamental principle of Granular Computing is that of an information granularity distribution and its optimization process. It has been used in system modelling to elevate a numerical model to its granular counterpart, which is more in rapport with reality. For example, in decision-making with fuzzy preference relations, it has been applied to enhance the existing numerical consistency improvement procedures. However, even though different protocols of information granularity distribution have been proposed, only the one based on a uniform and symmetric distribution and the one based on a symmetric but non-uniform distribution have been considered. Given that there exist others, this study aims to analyze how we can take advantage of all of them to improve the consistency of the fuzzy preference relations. Some numerical experiments are also completed to show the performance of these protocols.**

***Index Terms*—consistency, decision-making, fuzzy preference relation, information granularity**

I. Introduction

Decision-making consists in an individual or a group of them taking part to decide on the most suitable alternative between a set of them [1]. The most suitable alternative is the one that achieves the highest preference degree according to the individuals' viewpoints. Hence, the modeling of the individuals' viewpoints plays a pivotal role in decision-making. Between the existing ways of doing it, the pairwise comparisons in the form of preference relations have been demonstrated as a useful tool because they facilitates the aggregation of individual judgments into group ones and allow more precise assessments [2]. However, they could exhibit some contradictions [3]. It is obvious that decisions adopted based on contradictory (inconsistent) opinions must be avoided to ensure the validity of the outcome. Because consistency has been perceived as a measure of rationality [4], it allows degrees of performance, which implies the reliability of a preference relation can be measured by a consistency degree [5]. Numerous procedures attempting to improve consistency have been constructed [6]. In most of them, the improvement of consistency is achieved at the expense of high divergences between the original preference relation and the altered one, which is the cause of an information loss.

This issue has recently been resolved by integrating some restrictions into the improvement consistency process through the distribution of an average level of information granularity [7], [8]. The idea is to model the pairwise comparisons through information granules that make available a flexibility degree that can be exploited to improve the consistency. Conforming to it, different granular procedures have been developed to improve consistency [8]–[10]. Their common feature is the use of a uniform and symmetric distribution of information granularity. Although these procedures improve the consistency while restrict the range in which the pairwise comparisons are modified, it has been demonstrated that the flexibility and performance of these procedures is enhanced by an optimal (non-uniform) information granularity distribution [11]–[13].

There exist however unresolved issues to be addressed. Besides a symmetric information granularity distribution (both uniform and non-uniform) we can find other protocols of information granularity distribution [14]. The objective of this study is to describe how we can improve the consistency of the fuzzy preference relations with the existing protocols of information granularity distribution and their ensuing optimization.

This study is arranged as follows. Background knowledge is offered in Section II. Section III gives a description of

This work was supported by the grant PID2019-103880RB-I00 funded by MCIN / AEI / 10.13039/501100011033, and by the grant PID2022-139297OB-I00 funded by MCIN / AEI / 10.13039/501100011033 and by "ERDF A way of making Europe."

the existing protocols of information granularity distribution and their ensuing optimization to improve the consistency of a fuzzy preference relation. Numerical experiments are completed in Section IV to show the essence, flexibility and performance of the protocols. To conclude, Section V points out conclusions and future research directions.

II. Preliminaries

First, the decision-making process with fuzzy preference relations is described. Second, the consistency concept in fuzzy preference relations is presented. Finally, the basics of the particle swarm optimization (PSO) are recalled.

A. Fuzzy Decision-Making

Let a_i be an alternative, with $i = 1, 2, \ldots, m$. A decision-making process aims to produce an ordering of the alternatives based on the assessments provided by an individual or a group of them. The higher the position of an alternative in this ordering, the more suitable the alternative as solution to the decision-making process [1].

A preference relation modeling pairwise comparisons is the most used structure of preference elicitation in decision-making. However, to model the pairwise comparison, a domain of representation must also be established [16]. In this research, fuzzy preference relations are assumed, i.e., preference relations based on values in-between $[0, 1]$ to model the pairwise comparisons. We assume them because they have widely been used in decision-making and most of the granular procedures improving consistency are based on them [9], [11].

Definition 2.1: [15] "A fuzzy preference relation, F, is given by its membership function $\mu_F : A \times A \rightarrow [0, 1]$, where A is the set of alternatives."

The matrix $F = [f_{ij}]$ has typically been used to model the fuzzy preference relation F. In this matrix, each component $f_{ij} = \mu_F(a_i, a_j)$ is such that the higher f_{ij}, the higher the individual's preference of a_i over a_j: from $f_{ij} = 1$ denoting an absolute preference of a_i over a_j, through $f_{ij} = 0.5$ denoting indifference between the alternatives, to $f_{ij} = 0$ denoting an absolute preference of a_j over a_i. Due to the components of the leading diagonal, i.e., f_{ii}, are not taken into account, they are symbolized as "–" [15].

B. Consistency

Handling a fuzzy preference relation involves to bear in mind three rationality levels [17]: (i) Indifference is mandatory between an alternative and itself, (ii) a_i must be preferred to a_j if a_j is not preferred to a_i (reciprocity property), and (iii) a_i must be preferred to a_j if a_k is preferred to a_j and a_i is preferred to a_k (transitivity property). The only rationality level guaranteed by the formulation of a fuzzy preference relation is the first one. In fact, as mentioned, the preference of an alternative over itself is not taken into account. Nonetheless, to give consideration to a fuzzy preference relation as consistent, it must fulfill all the rationality levels, which has given rise to a number of properties that should be fulfilled by a fuzzy preference relation [18]. Between these properties, the additive transitivity has widely been used because it makes easy the verification of the consistency [6]. In [19], a procedure based on this property was built to quantify the consistency degree of a fuzzy preference relation. It refers to F as "additive consistent" if for every three alternatives, a_i, a_j, a_k, the values f_{ij}, f_{jk}, and f_{ik} satisfy:

$$f_{ik} = f_{ij} + f_{jk} - 0.5 \tag{1}$$

To measure the consistency, this procedure estimates f_{ik} $(i \neq k)$ by means of an intermediate alternative a_j and (1):

$$ef_{ik}^j = f_{ij} + f_{jk} - 0.5 \tag{2}$$

Then, the average of all ef_{ik}^j estimates the value of f_{ik} that is denoted as ef_{ik}:

$$ef_{ik} = \frac{1}{m-2} \sum_{j=1; j \neq i,k}^{m} ef_{ik}^j \tag{3}$$

The preference degrees present an absolute consistency if $ef_{ik}^j = f_{ik}\ \forall j$. Nonetheless, in real-world situations, it hardly happens. Consequently, the assessments could not satisfy (1). In [19], it was shown that the value that can be assigned to ef_{ik}^j is located in $[-0.5, 1.5]$. Therefore, to normalize the expression domain, the final estimated value of f_{ik} $(i \neq k)$, referred to as cf_{ik}, was computed as:

$$cf_{ik} = med\{0, 1, ef_{ik}\} \tag{4}$$

where med designates the median function.

The error between f_{ik} and cf_{ik}, referred to as ϵf_{ik}, is obtained as:

$$\epsilon f_{ik} = |cf_{ik} - f_{ik}| \tag{5}$$

Considering these values, the consistency degree of F, referred to as cd (the higher the value of cd, the higher the consistency degree of F), is computed as [19]:

$$cd = \frac{1}{m^2 - m} \sum_{i,k=1; i \neq k}^{m} (1 - \epsilon f_{ik}) \tag{6}$$

C. PSO Algorithm

PSO is a bio-inspired technique that emulates the movement and intelligence of a swarm to solve an optimization task [20]. It is an iterative process where the particles of the swarm work together in an intelligent way by exploring and exploiting a d-dimensional search space for locating the optimal solution according to a given quality measure (fitness function g). For a comprehensive explanation of this algorithm and its variants refer to [20] and [21].

In this study, we use the generic version, where the particle's velocity, $\mathbf{v}_i = (v_{i,1}, \ldots, v_{i,d})$, is updated according to $\mathbf{v}_i(\mathrm{t}+1) = \omega \mathbf{v}_i(\mathrm{t}) + c_1 \mathbf{r}(\mathbf{xl}_i - \mathbf{x}_i) + c_2 \mathbf{s}(\mathbf{xg} - \mathbf{x}_i)$, where $\mathbf{r} = (r_1, \ldots, r_d)$ and $\mathbf{s} = (s_1, \ldots, s_d)$ are two vectors of random numbers obtained from the uniform distribution over $[0, 1]$, and "t" designates the index of the current generation (iteration). In terms of the coefficients, c_1 and c_2 are two acceleration coefficients that influence the step size taken by the particle i towards its best position, $\mathbf{xl}_i = (xl_{i,1}, \ldots, xl_{i,d})$, and the global best position, $\mathbf{xg} = (xg_1, \ldots, xg_d)$, respectively, and ω stands for the inertia weight [20], [21]. The next position of the particle is obtained according to $\mathbf{x}_i(\mathrm{t}+1) = \mathbf{x}_i(\mathrm{t}) + \mathbf{v}_i(\mathrm{t}+1)$. We use this algorithm because its implementation is very easy, it has few parameters to adjust, and its convergence is very fast [21].

III. Improving Consistency by Distributing an Average Level of Information Granularity

The improvement of the consistency of a fuzzy preference relation entails that the individual must not have objection to modify the provided preferences. It calls for a specific flexibility degree that the individual must accept and that can be modeled by the information granularity concept [8]. Briefly, to facilitate the consistency improvement, the pairwise comparisons should be considered as granular realizations instead of precise numerical values in-between $[0, 1]$. The advantage of using the notion of information granularity is that it controls the difference between the initial preferences and the modified ones to some extent [8].

The existing granular procedures for improving consistency have modeled the granular realizations as intervals, and an information granularity level has been used to determine the length of the intervals. It means that an interval-valued preference relation, $IV(F)$, is built ($IV(\cdot)$ symbolizes an interval-valued preference relation family). In short, the length of the intervals makes available a flexibility that has been exploited to optimize a given optimization criterion that, in this case, is related to the consistency.

The rest of this section discusses how the current protocols of information granularity distribution, and their ensuing optimization, can be applied to improve the consistency. The protocols discussed are: (i) a uniform and symmetric information granularity distribution (s_1), (ii) a uniform but asymmetric information granularity distribution (s_2), (iii) a non-uniform but symmetric information granularity distribution (s_3), and (iv) a non-uniform and asymmetric information granularity distribution (s_4).

A. Uniform and Symmetric Distribution

Considering F, this protocol substitutes the numeric values of the entries by intervals that are distributed symmetrically around them and that have the same length. Let $\varepsilon \in [0, 1]$ be the average level of information granularity allowed by the individual, then we focus on each entry f_{ij}, whose value is modified within this interval:

$$iv_{ij} = [\max(0, f_{ij} - 0.5\varepsilon), \min(1, f_{ij} + 0.5\varepsilon)] \tag{7}$$

Let $\overline{F} \in IV(F)$, then this optimization model can be established:

$$\begin{cases} \max\limits_{\overline{f}_{ij}} cd \\ \text{s.t.} \begin{cases} |\overline{f}_{ij} - f_{ij}| < 0.5\varepsilon \\ \overline{f}_{ij} \in [0, 1] \end{cases} \end{cases} \tag{8}$$

The PSO algorithm is employed to resolve model (8), which returns the modified (optimal) fuzzy preference relation $\overline{F}$ maximizing the consistency. In this case, the dimension d of the particles is $m^2 - m$, which corresponds to the number of decision variables in model (8). The constraints are managed by the next expression that converts the value of each component of the particle, $x_{i,h} \in [0, 1]$, to its corresponding value within the admitted interval:

$$\overline{f}_{ij} = y + (z - y)x_{i,h} \tag{9}$$

where z and y are the upper and lower boundaries of the interval iv_{ij}.

B. Uniform but Asymmetric Distribution

Comparing to the previous protocol, this one provides more flexibility because, even though it is based on intervals with equal length, the intervals are asymmetrically allocated around the numeric value contained in each element of F. This asymmetric distribution brings a higher flexibility level that can be exploited during the optimization process. Considering f_{ij}, its value is modified within:

$$iv_{ij} = [\max(0, f_{ij} - \gamma_{ij}\varepsilon), \min(1, f_{ij} + (1 - \gamma_{ij})\varepsilon)] \tag{10}$$

where $\gamma_{ij} \in [0, 1]$ controls the asymmetric location of the interval related to f_{ij} and whose length is determined by the average level of information granularity ε. It means that different asymmetric locations of the intervals are allowed from one component to other, which increases the available flexibility level.

Let $\overline{F} \in IV(F)$, then this optimization model can be established:

$$\begin{cases} \max\limits_{\overline{f}_{ij}} cd \\ \text{s.t.} \begin{cases} |\overline{f}_{ij} - f_{ij}| < \varepsilon \\ \overline{f}_{ij} \in [0, 1] \end{cases} \end{cases} \tag{11}$$

The PSO algorithm is applied to resolve model (11), but different from protocol s_1, the constraints are managed as:

$$\overline{f}_{ij} = y + (z - y)x_{i,h} \tag{12}$$

where y and z are $\max(0, f_{ij} - \varepsilon)$ and $\min(1, f_{ij} + \varepsilon)$, respectively. According to it, through (10), we can compute the intervals where the values of the optimal fuzzy preference relation, $\overline{F}$, are set, i.e.:

$$\gamma_{ij} = \begin{cases} \dfrac{\varepsilon - |f_{ij} - \overline{f}_{ij}|}{2\varepsilon}, & \text{if } f_{ij} \leq \overline{f}_{ij} \\ 1 - \dfrac{\varepsilon - |f_{ij} - \overline{f}_{ij}|}{2\varepsilon}, & \text{otherwise} \end{cases} \tag{13}$$

C. Non-uniform but Symmetric Distribution

Considering F, this protocol substitutes the numeric values contained in its entries by intervals distributed symmetrically around them. However, in this case, each interval could have a different length. As a consequence, each f_{ij} is associated with a different information granularity level ε_{ij}. Considering f_{ij}, its value is modified within:

$$iv_{ij} = [\max(0, f_{ij} - 0.5\varepsilon_{ij}), \min(1, f_{ij} + 0.5\varepsilon_{ij})] \tag{14}$$

Let $\overline{F} \in IV(F)$, then this optimization model can be established:

$$\begin{cases} \max\limits_{\overline{f}_{ij}} cd \\ \text{s.t.} \begin{cases} \sum\limits_{i=1}^{m} \sum\limits_{j=1;j\neq i}^{m} 2|f_{ij} - \overline{f}_{ij}| \leq (m^2-m)\varepsilon \\ \overline{f}_{ij} \in [0,1] \end{cases} \end{cases} \quad (15)$$

where ε is the average level of information granularity allowed by the individual, i.e.:

$$\varepsilon = \frac{1}{m^2-m} \sum_{i=1}^{m} \sum_{j=1;j\neq i}^{m} \varepsilon_{ij} \quad (16)$$

Through model (15), the valued assigned to $\overline{f}_{ij}$ is calculated. After that, the value assigned to ε_{ij} is obtained. Let $\Delta_1 = (m^2-m)\varepsilon - \sum_{i=1}^{m}\sum_{j=1;j\neq i}^{m} 2|f_{ij} - \overline{f}_{ij}|$, then, ε_{ij} is allocated as:

$$\varepsilon_{ij} = \begin{cases} 2|f_{ij} - \overline{f}_{ij}| + \frac{1}{\#\Upsilon_1}\Delta_1, & \text{if } (i,j) \in \Upsilon_1 \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

where $\Upsilon_1 = \{(i,j) \mid \overline{f}_{ij} \neq f_{ij}\}$ and $\#\Upsilon_1$ is its cardinality. If $\Delta_1 = 0$, then the unique solution is $\varepsilon_{ij} = 2|\overline{f}_{ij} - f_{ij}|$. The PSO algorithm is applied to resolve model (15). The dimension d of the particles is also $m^2 - m$, which is the number of decision variables in model (15). Because $x_{i,h}(t) \in [0,1]$, $h = 1, 2, \ldots, d$, to manage the constraints in model (15), the fitness function g of the PSO is defined as:

$$g(\mathbf{x}_i(t)) = \begin{cases} cd, & \text{if } e(\mathbf{x}_i(t)) = 1 \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

where cd is the value of the consistency achieved considering the fuzzy preference relation generated by the vector received as parameter and computed using (6), and e is a function that controls that the information granularity level injected is lower than the one allowed by the individual. If the modified fuzzy preference relation constructed through the vector received as parameter has an average information granularity level higher than the one allowed, this function returns 0 as the fuzzy preference relation is not valid. If not, it returns 1.

Using the PSO, the entries of the fuzzy preference relation are modified to maximize the value of cd. Then, using (17), the value of every ε_{ij} is obtained.

D. Non-uniform and Asymmetric Distribution

This protocol replaces the numeric values contained in F by intervals with different length that are distributed asymmetrically around them. Considering f_{ij}, its value is modified within:

$$iv_{ij} = [\max(0, f_{ij} - \gamma_{ij}\varepsilon_{ij}), \min(1, f_{ij} + (1-\gamma_{ij})\varepsilon_{ij})] \quad (19)$$

where $\gamma_{ij} \in [0,1]$ controls the asymmetric location of the interval associated with f_{ij} and whose length is ε_{ij}.

Let $\overline{F} \in IV(F)$, then this optimization model can be established:

$$\begin{cases} \max\limits_{\overline{f}_{ij}} cd \\ \text{s.t.} \begin{cases} \sum\limits_{i=1}^{m} \sum\limits_{j=1;j\neq i}^{m} |f_{ij} - \overline{f}_{ij}| \leq (m^2-m)\varepsilon \\ \overline{f}_{ij} \in [0,1] \end{cases} \end{cases} \quad (20)$$

where ε is the average information granularity level allowed by the individual (see (16)).

Through model (20), we obtain the value that corresponds to $\overline{f}_{ij}$. After that, we calculate the value assigned to ε_{ij}. Let $\Delta_2 = (m^2-m)\varepsilon - \sum_{i=1}^{m}\sum_{j=1;j\neq i}^{m} |f_{ij} - \overline{f}_{ij}|$, then, ε_{ij} is distributed as:

$$\varepsilon_{ij} = \begin{cases} |f_{ij} - \overline{f}_{ij}| + \frac{1}{\#\Upsilon_2}\Delta_2, & \text{if } (i,j) \in \Upsilon_2 \\ 0, & \text{otherwise} \end{cases} \quad (21)$$

where $\Upsilon_2 = \{(i,j) \mid \overline{f}_{ij} \neq f_{ij}\}$ and $\#\Upsilon_2$ is its cardinality. If $\Delta_2 = 0$, then the unique solution is $\varepsilon_{ij} = |\overline{f}_{ij} - f_{ij}|$.

Using the PSO as in protocol s_3, the entries of the fuzzy preference relations are modified to maximize the value of cd. Then, through (21), the value of every ε_{ij} is obtained. Finally, by using (19), we can compute the intervals where the values of the optimal fuzzy preference relation, $\overline{F}$, are located, i.e.:

$$\gamma_{ij} = \begin{cases} \frac{\varepsilon_{ij} - |f_{ij} - \overline{f}_{ij}|}{2\varepsilon_{ij}}, & \text{if } f_{ij} \leq \overline{f}_{ij} \\ 1 - \frac{\varepsilon_{ij} - |f_{ij} - \overline{f}_{ij}|}{2\varepsilon_{ij}}, & \text{otherwise} \end{cases} \quad (22)$$

IV. Numerical Experiments

Let us suppose the following fuzzy preference relation containing the preferences expressed by an individual on five alternatives ($m = 5$):

$$F = \begin{bmatrix} - & 0.98 & 0.28 & 0.65 & 0.20 \\ 0.09 & - & 0.95 & 0.81 & 0.80 \\ 0.98 & 0.00 & - & 0.11 & 0.98 \\ 0.29 & 0.42 & 0.94 & - & 0.01 \\ 0.62 & 0.94 & 0.17 & 0.84 & - \end{bmatrix}$$

Through (6), the consistency degree of F is 0.653. To increase this value, we make use of the distribution of the information granularity along with its optimization. Concretely, we apply the four protocols described in Section III. Apropos of the average information granularity level ε, we assume a value of 0.1. Regarding to the PSO, after intense experimentation, it is executed with the following values: $\omega = 0.2$; $c_1 = c_2 = 2$; the number of generations is 100; and the swarm size is $10(m^2 - m)$. In what follows, we present the results returned by each protocol and investigate their performance for diverse values of the average information granularity level.

A. Uniform and Symmetric Distribution

For each component of F, this protocol builds intervals whose length is 0.1, being symmetrically distributed around the value given by the individual. As an example, for f_{13}, the interval built is $[0.23, 0.33]$. Considering it, the PSO produces:

$$\overline{F} = \begin{bmatrix} - & 0.93 & 0.32 & 0.69 & 0.24 \\ 0.13 & - & 0.90 & 0.76 & 0.75 \\ 0.93 & 0.00 & - & 0.15 & 0.93 \\ 0.33 & 0.46 & 0.89 & - & 0.05 \\ 0.57 & 0.89 & 0.21 & 0.79 & - \end{bmatrix}$$

Through (6), the consistency degree of $\overline{F}$ is 0.696. In this case, because all the intervals have the same length and are allocated symmetrically around the initial value given by the individual, we do not show neither the information granularity level distributed nor the intervals constructed.

B. Uniform but Asymmetric Distribution

For each component of F, this protocol builds intervals whose length is 0.1, but, different from the above protocol, they are asymmetrically distributed around the value provided by the individual. Considering it, the PSO generates:

$$\overline{F} = \begin{bmatrix} - & 0.88 & 0.37 & 0.74 & 0.29 \\ 0.18 & - & 0.85 & 0.71 & 0.70 \\ 1.00 & 0.00 & - & 0.20 & 0.88 \\ 0.38 & 0.51 & 0.84 & - & 0.10 \\ 0.56 & 0.84 & 0.26 & 0.74 & - \end{bmatrix}$$

Through (6), the consistency degree of $\overline{F}$ is 0.733. This protocols distributes the same information granularity level to each component of F, 0.1 in this case. Related to the intervals, IV contains the intervals built for each component of $\overline{F}$:

$$IV = \begin{bmatrix} - & [0.88, 0.98] & [0.275, 0.375] & [0.645, 0.745] & [0.195, 0.295] \\ [0.085, 0.185] & - & [0.85, 0.95] & [0.71, 0.81] & [0.70, 0.80] \\ [0.90, 1.00] & [0.00, 0.10] & - & [0.105, 0.205] & [0.88, 0.98] \\ [0.285, 0.385] & [0.415, 0.515] & [0.84, 0.94] & - & [0.005, 0.105] \\ [0.54, 0.64] & [0.84, 0.94] & [0.165, 0.265] & [0.74, 0.84] & - \end{bmatrix}$$

C. Non-uniform but Symmetric Distribution

For each component of F, this protocol builds intervals of different length symmetrically distributed around the value provided by the individual. Because $\varepsilon = 0.1$, it means that the average of the length of the intervals must be 0.1. Considering it, the PSO returns this optimal fuzzy preference relation:

$$\overline{F} = \begin{bmatrix} - & 0.76 & 0.27 & 0.65 & 0.20 \\ 0.20 & - & 0.94 & 0.80 & 0.79 \\ 0.97 & 0.07 & - & 0.17 & 0.95 \\ 0.29 & 0.41 & 0.68 & - & 0.02 \\ 0.62 & 0.93 & 0.33 & 0.83 & - \end{bmatrix}$$

Through (6), the consistency degree of $\overline{F}$ is 0.722. Based on this optimal fuzzy preference relation, the matrix, D, is constructed, which contains the information granularity distributed to each entry of $\overline{F}$:

$$D = \begin{bmatrix} - & 0.44 & 0.02 & 0.00 & 0.00 \\ 0.22 & - & 0.02 & 0.02 & 0.02 \\ 0.02 & 0.14 & - & 0.12 & 0.06 \\ 0.00 & 0.02 & 0.52 & - & 0.02 \\ 0.00 & 0.02 & 0.32 & 0.02 & - \end{bmatrix}$$

Considering the information granularity level distributed to each component, the matrix IV containing the intervals built for each component of $\overline{F}$ is the following:

$$IV = \begin{bmatrix} - & [0.76, 1.00] & [0.27, 0.29] & [0.65, 0.65] & [0.20, 0.20] \\ [0.00, 0.20] & - & [0.94, 0.96] & [0.80, 0.82] & [0.79, 0.81] \\ [0.97, 0.99] & [0.00, 0.07] & - & [0.05, 0.17] & [0.95, 1.00] \\ [0.29, 0.29] & [0.41, 0.43] & [0.68, 1.00] & - & [0.00, 0.02] \\ [0.62, 0.62] & [0.93, 0.95] & [0.01, 0.33] & [0.83, 0.85] & - \end{bmatrix}$$

D. Non-uniform and Asymmetric Distribution

For each component of F, this protocol builds intervals of different length asymmetrically distributed around the value provided by the individual. As in the previuos protocol, because $\varepsilon = 0.1$, it means that the average of the length of the intervals must be 0.1. Considering it, the PSO generates the following optimal fuzzy preference relation:

$$\overline{F} = \begin{bmatrix} - & 0.51 & 0.29 & 0.65 & 0.20 \\ 0.38 & - & 0.94 & 0.80 & 0.79 \\ 0.69 & 0.29 & - & 0.38 & 0.81 \\ 0.29 & 0.41 & 0.88 & - & 0.01 \\ 0.62 & 0.93 & 0.25 & 0.82 & - \end{bmatrix}$$

Through (6), the consistency degree of $\overline{F}$ is 0.781. Based on this optimal fuzzy preference relation, the matrix, D, is constructed, which contains the information granularity distributed to each entry of $\overline{F}$:

$$D = \begin{bmatrix} - & 0.47 & 0.01 & 0.00 & 0.00 \\ 0.29 & - & 0.01 & 0.01 & 0.01 \\ 0.29 & 0.29 & - & 0.27 & 0.17 \\ 0.00 & 0.01 & 0.06 & - & 0.00 \\ 0.00 & 0.01 & 0.08 & 0.02 & - \end{bmatrix}$$

Considering the information granularity level distributed to each component, the matrix IV containing the intervals built for each component of $\overline{F}$ is the following:

$$IV = \begin{bmatrix} - & [0.51, 0.98] & [0.28, 0.29] & [0.65, 0.65] & [0.20, 0.20] \\ [0.09, 0.38] & - & [0.94, 0.95] & [0.80, 0.81] & [0.79, 0.80] \\ [0.69, 0.98] & [0.00, 0.29] & - & [0.11, 0.38] & [0.81, 0.98] \\ [0.29, 0.29] & [0.41, 0.42] & [0.88, 0.94] & - & [0.01, 0.01] \\ [0.62, 0.62] & [0.93, 0.94] & [0.17, 0.25] & [0.82, 0.84] & - \end{bmatrix}$$

E. Comparative Analysis

Although the average level of information granularity allowed has been low ($\varepsilon = 0.1$), the results achieved by each protocol show that a distribution of information granularity along with its optimization can significantly improve the consistency compared to when precise numerical values are utilized, i.e., the numerical procedure developed in [19], which gets a consistency degree of 0.653, has been improved by its granular counterparts, s_1, s_2, s_3, and s_4, which achieve a value of 0.696, 0.733, 0.722, and 0.781, respectively. We have also conducted an experiment in which 100 fuzzy preference relations have randomly generated with different number of alternatives ($m = 4$, $m = 5$, $m = 6$, and $m = 7$). Assuming $\varepsilon = 0.1$, the results show that the consistency achieved by the procedure developed in [19] has been improved by its granular counterparts, s_1, s_2, s_3, and s_4, with an average percentage of 6.7%, 12.4%, 10.7%, and 19.8%, respectively.

Finally, using F and with the objective of investigating the impact of ε on the performance of the protocols, we execute them by setting ε to 0.2, and 0.3. Table I shows the evolution of the values for the consistency for different granule sizes. It seems easy to understand that the greater the ε injected into the model, the higher the probability of reaching higher values of consistency (cd). This can be observed in Table I, where a clear increasing trend in the values of cd is visible in line with the increment of the values of ε. This is due to the greater the value assigned to ε, the higher the length of the intervals in which the

TABLE I
VALUES OF cd FOR CHOSEN VALUES OF ε

	$\varepsilon = 0.1$	$\varepsilon = 0.2$	$\varepsilon = 0.3$
s_1	0.696	0.733	0.781
s_2	0.733	0.832	0.914
s_3	0.722	0.780	0.893
s_4	0.781	0.895	0.978

preference degree can be located and, therefore, the possibility of maximizing the consistency is greater. On the other hand, independently of the average information granularity level allowed, the protocol based on an asymmetric and non-uniform distribution of information granularity is the one that achieves greater values of the consistency. It seems natural because, between all the protocols discussed, this is the one exhibiting the highest flexibility level.

V. CONCLUSIONS

This study has discussed four protocols of information granularity distribution to improve the consistency degree of a fuzzy preference relation. While there have been other studies, concretely those coming from a symmetric distribution of information granularity (both uniform and non-uniform), a complete study considering all the protocols has not been completely performed. The numerical experiments carried out has shown the effectiveness and performance of the protocols. Comparing to the protocols based on a symmetric distribution, those based on an asymmetric distribution have reached greater values of consistency. Between them, the protocol based on a non-uniform distribution of information granularity has reached the best value. The reason is that this protocol is the most flexible, which allows for a more effective usage of the resources by allocating higher information granularity levels to the entries that require it the most.

As a first step at attempting to improve this study, the next research directions emerges. Because PSO (metaheuristic) has been used to obtain the modified fuzzy preference relation improving the consistency, a statistical analysis is critical in examining if the comparative results presented in Section IV-E are conclusive or not. On the other hand, the need for a two-objective optimization becomes apparent in case of group decision-making. Here, besides the consistency, the consensus [22], [23], i.e., the agreement between the individuals, could also be maximized.

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