2023 IEEE Symposium Series on Computational Intelligence (SSCI)
Mexico City, Mexico. December 5-8, 2023

Optimal Allocation of PV Systems on Unbalanced Networks Using Evolutionary Algorithms

Wenlei Bai
Oracle Energy and Water
Oracle Corporation
Austin, TX, USA
baiwenlei123@gmail.com

Wen Zhang
Business School
Baylor University
Waco, TX, USA
wen_zhang1@baylor.edu

Fanlin Meng
Business School
University of Manchester
Manchester, UK
fanlin.meng@manchester.ac.uk

Richard Allmendinger
Business School
University of Manchester
Manchester, UK
richard.allmendinger@manchester.ac.uk

Kwang Y. Lee
Electrical & Computer Engineering
Baylor University
Waco, TX, USA
kwang_y_lee@baylor.edu

Abstract—**As the distributed energy resources (DERs) increasingly penetrate the unbalanced distribution network, it becomes challenging to accommodate such penetration technically and economically. Therefore, this paper tackles an optimal allocation of PV systems (locations and sizes) to maximize the penetration while minimizing voltage violation. It is challenging because the problem is a mixed integer nonlinear programming (MINLP) problem with non-linear and non-convex properties. In addition, the network is unbalanced which brings burdens on solving load flows. Computational intelligent methods, particularly evolutionary algorithms (EAs) have proven its efficiency and robustness in large optimization problems and thus, this paper explores two EAs on the problem with the help of a robust unbalanced load flow algorithm. A comparative study is conducted on particle swarm optimization (PSO) and artificial bee colony (ABC) based on IEEE 13 and 37 bus systems. Optimal allocation based on peak hour and day-ahead scenarios are considered. After 30 times run, the test cases have shown that both EAs are successful and yet ABC generally converges to better solution and yet with larger statistical deviations on solutions.**

Keywords— Artificial bee colony; Distributed energy resources; Evolutionary algorithms; Particle swarm optimization PV allocation; Unbalanced network.

I. Introduction

The high penetration of distributed energy resources (DERs) into distribution network has positive impacts on environment, system reliability and flexibility, and yet it also brings challenges from operation and planning perspectives. Today, the distribution network has become more active in the sense of exchanging energy, decentralized control locally in nearly real time, and due to the continuously developing distributed energy resource (DER) technologies and information and communication technologies (ICT) [1]. The IEEE 1547-2018 standard has come out to assist the high penetration of DERs in 2018, which also indicated a new era for DERs planning and management [2]. Photovoltaic (PV) system is one of the popular DERs to be considered by planners, thanks to its flexibility, scalability, low operational costs, and mitigation of demand peak [3]. Yet PV systems can also bring challenges, and one major challenge is that it can cause rapid voltage changes on the network, especially during sudden changes in solar irradiance. Maintaining voltage within acceptable limits becomes a challenge. Thus, optimal allocation, regarding PV systems' location and size, is critical for planners.

Distribution power flow was not invented until 1990s [4] and it's the fundamental tool to ensure the allocation of DERs will not violate operating limits such as node voltage and line current flow. Newton Raphson method is widely used in transmission network and adopted to distribution network recently [5]. Yet the computational cost is high due to inversion of Jacobian matrix in each iteration. Another popular well-known method, backward/forward sweep, is proposed in early 90s for balanced network [4]. One of the obvious drawbacks is that it only fits for balanced network. In this work, a simple yet efficient fixed-point method is adopted to tackle 3-phase unbalanced load flow. The fixed-point method models power conversion elements (generators, loads) as Norton current equivalent circuits such that node voltages can be solved iteratively with constant system admittance matrix. In other words, the matrix inversion is calculated only one time and it can solve very unbalanced network [6]. Details are explained in Section II.

The PV allocation problem essentially becomes a mixed integer nonlinear programming (MINLP), which is hard to solve with traditional mathematical approaches. Recently, computational intelligent methods have proven its effectiveness in complex engineering problems [7]-[9]. Janamala and Rani [10] implemented a recent meta-heuristics Archimedes Optimization Algorithm (AOA) to solve the optimal allocation problem. This work explores the two well-known EAs, artificial bee colony (ABC) and particle swarm optimization (PSO), to demonstrate their effectiveness.

In all, the contributions of this paper are:

1) The paper formulates a PV allocation optimization problem whose objective is to maximize PV systems penetration while not violating operating limits.
2) A simple yet efficient fix-point method is implemented to solve unbalanced power flow to

embrace the 3-phase unbalanced feature in distribution network.

3) A comparative study is conducted between ABC and PSO to demonstrate the effectiveness of EAs in complex MINLP.

The paper is organized as follows: Section II formulates the PV allocation problem and then followed by fixed-point method. Section III describes the methodology and implementation of ABC and PSO. Section IV presented the case study with two EAs and comparative analysis. Finally, Section V concludes the paper with recommendation for future works.

II. Problem Formulation

The PV system allocation problem is first introduced in this section, which is modelled as MINLP and then followed by the iterative load flow method.

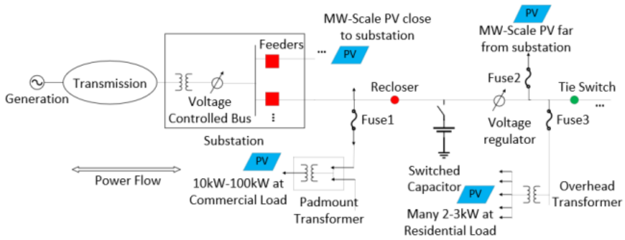

Fig. 1: High penetration of PV system.

A. PV Allocation

Figure 1 illustrates an overview of high PV system penetration into an unbalanced distribution network. It indicates that there are MW-scale, commercial and residential types of PV systems and the power flow is bidirectional. The PV allocation problem is to find the optimal location and sizing of PV systems to minimize voltage violation while maximizing power injection subject to certain equality and inequality constraints. The mathematical form is expressed as followings:

$$\min f(u) \tag{1a}$$

$$g(u,x,y) \le 0 \tag{1b}$$

$$h(u,x,y) \tag{1c}$$

where u is the control variable including PV locations and size, x is the state variable/dependent variable including voltages and angles at each bus, y is the known network parameters such as network resistance, impendence, device rating, etc.; $g(\cdot)$ is the inequality constraints which include line flow limit, voltage limit, PV active power injection limit; $h(\cdot)$ is the equality constraints which is the power balance equation at each node (highly nonlinear equations). Details can be found in [11].

For the objective function $f(\cdot)$, two objectives, f_1 and f_2, are considered in the study, where f_1 is to minimize voltage violation while maximizing power injection at specific hour; f_2 is to minimize voltage violation while maximizing power injection in 24-hour horizon as shown below:

$$f_1 = \sum_{i=1}^{N_{PV}} \begin{cases} Pen(V_L - V_i) - P_i, & V_i < V_L \\ -P_i, & V_L < V_i < V_H \\ Pen(V_i - V_H) - P_i, & V_i > V_H \end{cases} \tag{2a}$$

$$f_1 = \sum_{t=1}^{24} \sum_{i=1}^{N_{PV}} \begin{cases} Pen(V_L - V_i) - P_i, & V_i < V_L \\ -P_i, & V_L < V_i < V_H \\ Pen(V_i - V_H) - P_i, & V_i > V_H \end{cases} \tag{2b}$$

where Pen is the penalty factor; V_L and V_H are the low and high limits for voltage; V_i and P_i are respectively the bus voltage and the real power injection at PV bus i.

B. Unbalanced Load Flow Method

Unlike the transmission network, distribution network is unbalanced and thus distribution load flow is the foundation for many advanced studies. Examples are 1) voltage quality analysis: size and locations of capacitor banks, locations and rating of voltage regulators, line upgrades, 2) DER integration: given location of new DER, determine impact on operations. 3) Outage restoration analysis, which is done by real-time power flow: if outage occurs, determine how to operate switches to restore load.

Popular methods such as backward/forward sweep and Newton Raphson have their drawbacks. For example, the drawbacks for backward/forward sweep include 1) Handling Distributed Generation: The method can have trouble dealing with systems that have high penetration of distributed generation sources. In this situation, the power flow might not be unidirectional, which violates an assumption of the method; 2) Computational Efficiency: While it's relatively efficient for smaller systems, the computational time can grow rapidly for large-scale, multi-phase, and unbalanced distribution systems.

The drawbacks for Newton Raphson method include 1) Jacobian Matrix Computation: The Jacobian matrix used in the method needs to be updated and inverted in each iteration, which is computationally intensive, particularly for large-scale power systems; 2) Radial Distribution Systems: The Newton-Raphson method is more suited to transmission systems (which are generally mesh networks) and can have trouble with the unique characteristics of distribution systems, which are typically radial and have more unbalanced loads.

Therefore, this work adopted an efficient iterative method called fixed-point method by OpenDSS [6]. The mathematical process is described as following:

$$I_{inj} = Y_{system} V \tag{3}$$

where I_{inj} is the compensation, or injection, currents vector at each node from power conversion elements (generator, loads) in the circuit, which may be nonlinear elements, not constant and depend on node voltage; V is the node voltage vector; Y_{system} is the admittance matrix constructed by Y_{prim}, as opposed to the admittance matrix in a transmission network, it contains the three-phase information; Y_{prim} is the primitive matrix for a particular element (line, load, generator, etc.) to present the admittance information between its terminals.

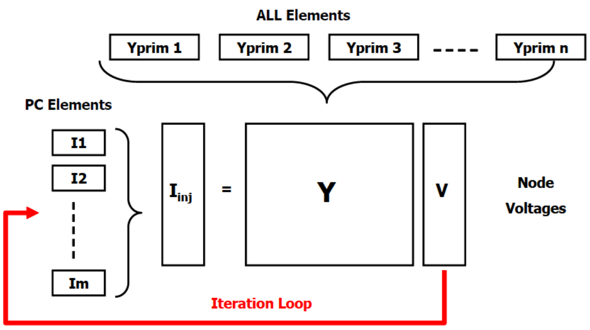

Fig. 2: Fixed-point method.

The process is straightforward. First, use initial guess of V_0 to calculate I_{inj} and then voltage is calculated iteratively as following until the algorithm converges (the difference in current and previous solution is within a predefined threshold):

$$V_{n+1} = Y_{system}^{-1} I_{inj}(V_n), \quad n = 0,1,2,\cdots,n \tag{4}$$

III. Methodology

Evolutionary computation (EC) has been proven in its effectiveness over complex optimization problems, and many of which are in energy domain [8][9]. In this work, two population-based evolutionary algorithms, ABC and PSO are implemented and compared to evaluate their effectiveness. The basic ABC and PSO have been proven effective on various complex engineering problems with a good balance of exploration and exploitation [9][13]. Thus, they are chosen as benchmark performance and future work will develop improved EAs to tackle the problem.

A. Solution Vector

EA's solution vector structure is demonstrated in Fig. 3. The control variables consist of location as discrete variable and size as continuous variable over 24-hour periods for day-ahead planning. The solution vector for peak hour planning just consists of location and size variables for 1-hour period.

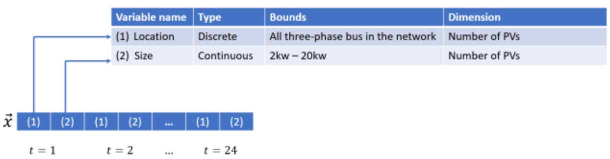

Fig. 3: EAs solution vector structure.

B. Artifical Bee Colony on PV Allocation

The ABC is a population-based search algorithm, mimicking the foraging of honeybees. Bees are sent out to randomly search in multidimensional feasible space (bounded by limits) to look for food sources (solutions) [12].

At initialization, each solution $U_i = \{U_{i,1}, U_{i,2}, \ldots, U_{i,D}\}$ is generated randomly within the limits of the variables as follows:

$$U_{i,j} = U_{i,j\_\min} + rand(0,1)(U_{i,j\_\max} - U_{i,j\_\min}) \tag{5}$$

where U_{i,j_min} and U_{i,j_max} are the lower and upper bounds for the j^{th} dimension for the i^{th} food source. Note that there is a total SN number of food sources, and D control variables, and $rand(0,1)$ is a random number in (0,1) obtained by uniform distribution.

Every food source will be updated to a new candidate solution based on their neighborhood's information. The nectar of new solutions (fitness value) will be evaluated to decide whether the current solution is to be replaced by the new one. Such selection is known as 'greedy selection.'

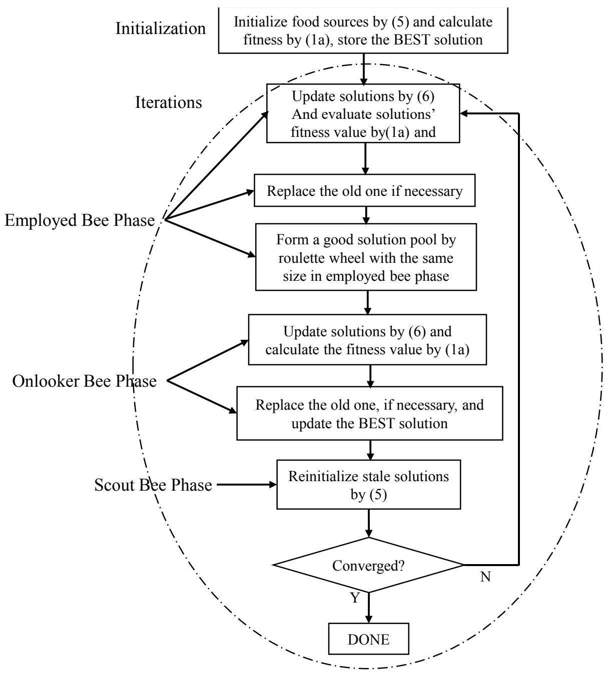

Fig. 4: The overall ABC structure.

The update equation for a new candidate solution vector V_i is defined as:

$$V_{i,j} = U_{i,j} + \Phi_{i,j}(U_{i,j} - U_{k,j}) \tag{6}$$

where k is a different integer other than i, randomly chosen from the size of employed bees (SN), and $\Phi_{i,j}$ is a random number from [-1,1].

C. Particle Swarm Optimization on PV allocation

The PSO is also a population-based searching algorithm, introduced by Kennedy and Eberhart in 1995, to explore the search space by particles [13]. One particle consists of velocity and position, and position is the feasible solution updated with the help of previous position and velocity as shown below:

$$V_i^{new} = W_I V_i + R_1 W_M (b_i - X_i) + R_2 W_C (b_G - X_i) \tag{7}$$

$$X_i^{new} = X_i + V_i^{new} \tag{8}$$

where W_I, W_M, W_C are weights for inertia, memory, and cooperation terms, R_1, R_2 are two random numbers from 0 to 1, b_i, b_G are personal best and global best, X_i^{new} is the new solution computed with the help of V_i^{new}. Figure 5 shows the flow chart of PSO on PV allocation.

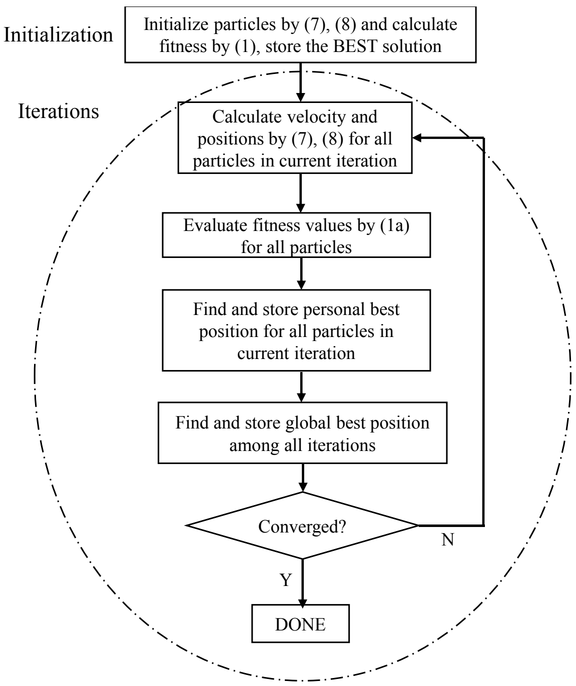

Fig. 5: The overall PSO structure.

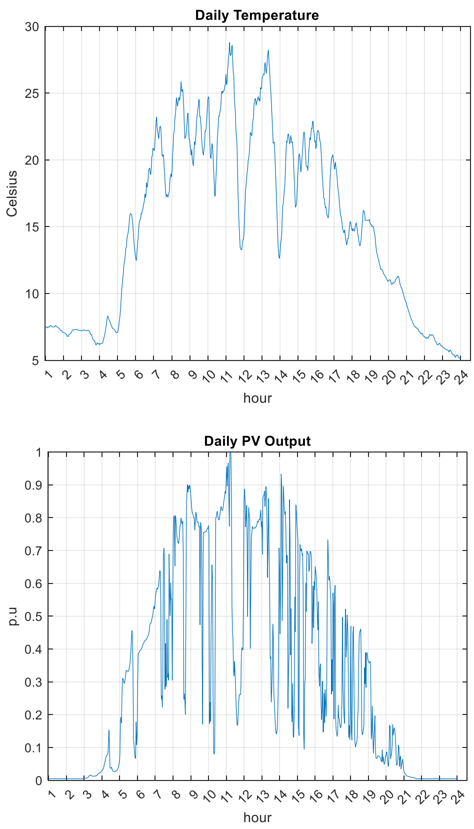

Fig. 7: Temperature and PV output.

IV. Results and Discussion

This section first describes case studies where one 3-phase PV is chosen to be installed on the IEEE 13 and 37 bus systems, and then followed by performance analysis and discussion. Note that the potential buses to install PV systems are the ones with three-phase 4.16 kV voltage level. Other buses are either in different voltage level or with unbalanced phases.

A. PV and Load Profile

For this work, a 3-phase PV system with rated 4.16 kV, possible output 2,000-20,000 kVA is chosen, and it is assumed with power factor = 1. Fig. 6 gives the load profile of the chosen date. Fig. 7 shows the temperature and PV output for the selected date. It is obvious that PV output has positive correlation with the temperature.

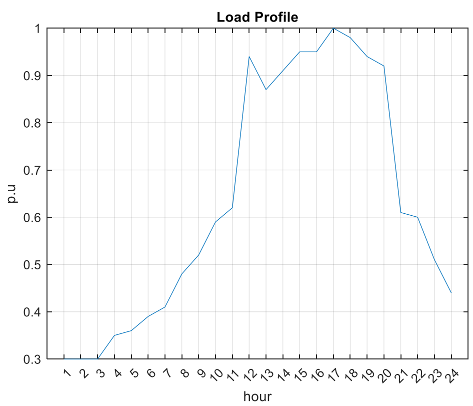

Fig. 6: load profile.

B. PV Allocation on IEEE 13 and 37 bus Test systems

In the case study on IEEE 13 bus system, the potential buses to interconnect PV systems are {670, 671, 633, 680, 675, 692} and the possible size is from 2,000 – 20,000 kVA as shown in Figure 8.

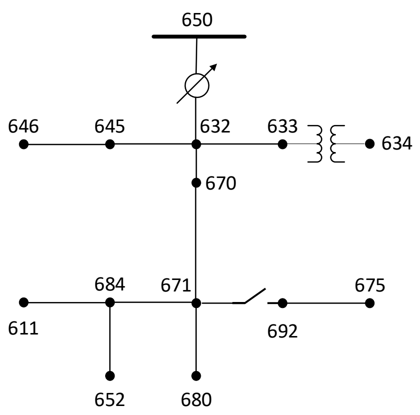

Fig. 8: IEEE 13 bus test system.

TABLE I. CASE STUDY ON IEEE-13 BUS

	Peak-hour planning (13 bus)						
	Min	Avg	Max	Std	T(s)	Solution (Bus, kVA)	#pop/iterations
ABC	-16791	-16788	-16785	1.5	5.1	(670, 16791)	100/200
PSO	-16791	-16790	-16786	1.1	6.3	(670, 16791)	100/200
	Day-ahead planning (13 bus)						
	Min	Avg	Max	Std	T(s)	Solution (Bus, kVA)	#pop/iterations
ABC	275663	303226	360556	1659	51	(670, 14069)	80/200
PSO	325933	325963	326835	164	38	(670, 12243)	60/200

As mentioned, there are two scenarios in the case study: 1) optimal location and sizing of 1 PV system at **12:00pm** (peak hour planning); 2) optimal location and sizing of 1 PV system over one day (day-ahead planning).

Table I shows the results for IEEE 13 bus system for 30 runs. Column 'Solution' means the minimum optimal result found by evolutionary algorithms among 30 runs, which shows the bus location and kVA injection size. Negative fitness value means the solution (location and size of PV system) has not led to voltage violation, and yet positive fitness value means the solution has introduced voltage violation in (2a) and (2b). It is noted that 'min' found by ABC is much less than that found by PSO in Day-ahead planning scenario and yet its standard deviation is much larger than that of PSO. The fact that both ABC and PSO cannot find solutions which do not result in voltage violation, implies that feasible solutions may not exist to penetrate a 3-phase PV systems over 24-hour periods without violating voltage limits for this network.

TABLE II. CASE STUDY ON IEEE-37 BUS

	Peak-hour planning (37 bus)						
	Min	Avg	Max	Std	T(s)	Solution (Bus, kVA)	#pop/iterations
ABC	-8939	-8549	-8151	165	21.1	(705, 8939)	100/200
PSO	-9167	-8984	-8270	272	22.2	(705, 9167)	100/200
	Day-ahead planning (37 bus)						
	Min	Avg	Max	Std	T(s)	Solution (Bus, kVA)	#pop/iterations
ABC	-8057	-7723	-7579	115	161.1	(705, 8057)	80/200
PSO	-7824	-7807	-7585	60	122.2	(714, 7824)	80/200

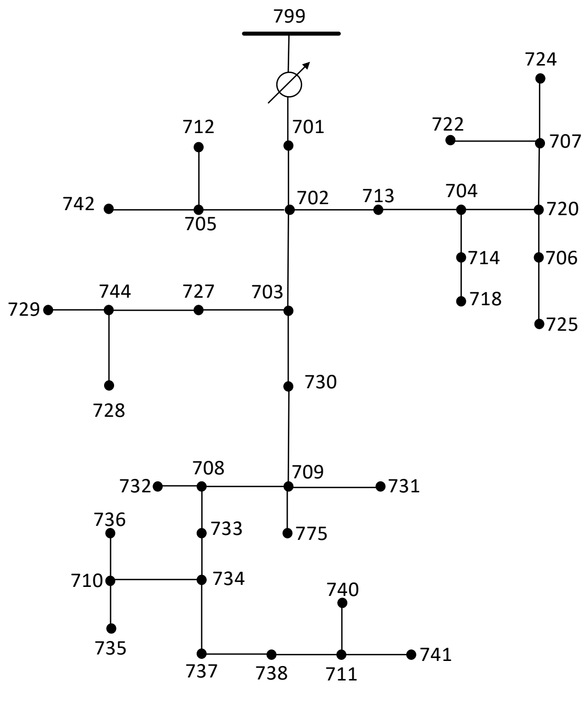

Fig. 9: IEEE 37 bus test system.

In next case study on IEEE 37 bus system as shown in Fig. 9, the potential buses to interconnect PV systems are {701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 718, 720, 722, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 740, 741, 742, 744} and the possible size is from 2,000 – 20,000 kVA. Similarly, two scenarios are conducted under the test system including peak hour planning and day-ahead planning.

Table II shows the results for IEEE 37 bus system for 30 runs. Column 'Solution' means the minimum optimal result found by EAs among 30 runs, which shows the bus location and kVA injection size. Then 'min' found by ABC is less than that found by PSO in Day-ahead planning scenario and yet its standard deviation is larger than that of PSO. Fitness values are all negative which means that solutions don't result in voltage violation for both scenarios. Optimal location found by ABC and PSO for Scenario 2 is different.

V. CONCLUSIONS

Distribution network has become active and complicated, containing bi-lateral power flow and large DERs penetration. Finding optimal location and size of PV systems is a complex planning problem and yet critical to distribution network management. This work explored the possibility of using two EAs (ABC and PSO) to tackle such problem with the help of a simple and yet efficient fixed-point iterative load flow method, which ensures the unbalanced network be solved successfully. Both EAs are verified on IEEE 13 and 37 bus systems with two objectives (peak-hour and day-ahead planning). After 30-time runs, both EAs are relatively successful in finding solutions, but they have different attributes such as the ability to find lower fitness values, and stable fitness values. The ABC generally converges with better solution and yet with statistically larger deviations on solutions. This work proves the EAs' efficiency on solving such problem and is a realistic tool for operators to plan PV systems' integration on unbalanced distribution network. Future work can focus on integrating multiple PV systems using basic EAs and/or improved EAs.

REFERENCES

[1] M. E. Honarmand, V. Hosseinnezhad, B. Hayes, and P. Siano, "Local energy trading in future distribution systems," *Energies*, vol. 14, pp. 3110-3129, May 2021.

[2] IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces. https://standards.ieee.org/ieee/1547/5915/ (accessed Aug. 8, 2023).

[3] R. O. Bawazir and N. S. Cetin, "Comprehensive overview of optimizing PV-DG allocation in power system and solar energy resource potential assessments," *Energy Report*, vol. 6, pp. 173-208, 2020.

[4] D. Rajicic, R. Ackovski, and R. Taleski, "Voltage Correction Power Flow," *IEEE Trans. Power Deliv*., vol. 9, no. 2, pp. 1056–1062, Apr. 1994.

[5] M. Karimi, A. Shahriari, M. R. Aghamohammadi, H. Marzooghi, and V. Terzija, "Application of Newton-based load flow methods for determining steady-state condition of well and ill-conditioned power systems: A review," *Int. J. of Electrical Power&Energy Systems*, vol. 113, pp. 298-309, 2019.

[6] R. C. Dugan and T. E. McDermott, "An open source platform for collaborating on smart grid research," 2011 *IEEE PES General Meeting*, Detroit, Michigan, U.S, July 24–28, 2011.

[7] W. Bai, X. Zhu, and K. Y. Lee, "Dynamic otpimal power flow based on a spatio-temporal wind speed forecast model," 2021 *IEEE Congress on Evolutionary Compuation*, Krakow, Poland, pp. 136-143.

[8] W. Bai, M. Abedi, and K. Y. Lee, "Distributed generation system control strategies with PV and fuel cell in microgrid operation," *Control Engineering Practice*, vol. 53, pp. 184-193, 2016.

[9] W. Bai, I. Eke, and K. Y. Lee, " An improved artificial bee colony optimization algorithm based on orthogonal learning for optimal power flow problem," *Control Engineering Practice*, vol. 61, pp. 163-172, 2017.

[10] V. Janamala and K. R. Rani, "Optimal allocation of solar photovoltaic distributed generation in electrical distribution networks using archimedes optimization alogrithm," *Clean Energy*, vol. 6, pp. 271-287, 2022.

[11] R. Fathi, B. Tousi, and S. Galvani, "A new approach for optimal allocation of photovoltaic and wind clean energy resources in distribution networks with reconfiguration considering uncertainty based on info-gap decision theory with risk aversion strategy," *Journal of Cleaner Production*, vol. 295, pp. 125984 – 126002, 2021.

[12] D. Karaboga, "An idea based on honey bee swarm for numerical optimization," *Technical Report TR06*, Erciyes University, Engineering Faculty, Computer Engineering Department, 2005.

[13] R. C. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," In*: Proceedings of the 6th international symposium on micro machine and human science*, pp 39–43, Nagoya, Japan, Mar 13–16, 1995.