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A Brief Review of Multi-concept Multi-objective Optimization Problems

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***Abstract*—In the context of design, multi-concept optimization (MCO) refers to the task of concurrently identifying the best concept and the corresponding variable values to optimize certain objective(s). Despite its relevance in various practical domains such as engineering, transport, and product design, there have been limited studies on developing computationally efficient algorithms specialized for MCO problems. One of the contributing factors towards this gap is the lack of benchmark problems for MCO that offer diverse challenges for the systematic evaluation and development of advanced algorithms. In this paper, we conduct a brief review of some existing multi-objective test problems in the domain and discuss some of their shortcomings. The key aim is to highlight the need for the development of a more extensive set of benchmark problems that are flexible and tunable in terms of the challenges posed to the solution methodologies. In turn, we hope that this will encourage the development of more advanced algorithms to solve practical MCO problems in the future.**

***Index Terms*—Multi-concept optimization, multi-objective optimization, multi-concept benchmark problems**

I. Introduction

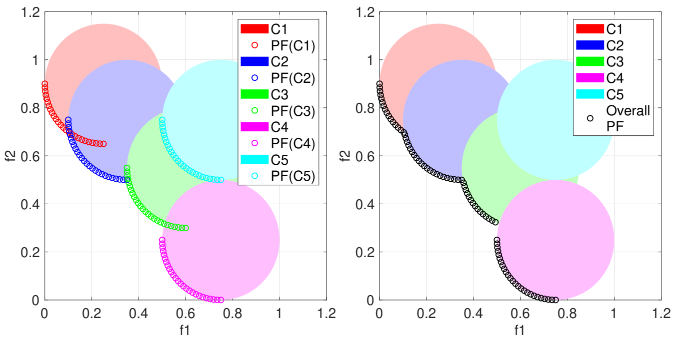
Optimization forms an integral part of the design process, wherein a systematic exploration of the controllable parameters is undertaken to improve certain performance objective(s). The applications span a diverse range of domains, such as engineering, finance, operations research, to name a few. The problems that involve only one objective are termed single-objective optimization problems (SOPs), whereas those that involve more than one conflicting objectives are referred to as multi-objective optimization problems (MOPs). For SOPs, the theoretical optimum consists of a single globally best objective value, which may be achieved through one or more solutions in the design space. For MOPs, however, given the conflict between the objectives, a unique solution (in objective space) is not possible. Rather, the theoretical optimum comprises a set of trade-off designs in the objective space referred to as the Pareto-optimal Front (PF). The corresponding solutions in the variable space are referred to as the Pareto-optimal Set (PS).

While various types of SOPs and MOPs have been widely studied in the literature, the focus has been predominantly on the problems where a given solution "concept" is already fixed. However, the solution process to real-world problems often begins with defining a number of plausible competing solution concepts in the initial stage and then choosing the most promising ones based on experience or preliminary analysis. The standard optimization algorithms typically operate on a fixed concept and are therefore generally suited to later stages, such as detailed design. On the other hand, they are not well-suited or efficient for application to the initial design stages, such as concept selection and preliminary design, which have a significant impact on the product life-cycle costs [1], [2].

In a bid to address the above gaps, a specialized category of problems has been defined that considers the concept selection and optimization simultaneously, referred to as Multi-Concept Optimization (MCO) [3]. In MCO problems, a number of potential solution concepts are defined, with their respective decision variables spaces. The target for a prospective solution method is to search through the space of concept as well as the corresponding variables concurrently to identify the best of both. To cite a few examples, three unit cell types were considered as different concepts for lattice design in [3], three different winglet concepts were considered for the winglet design in [4], six cross-sectional shapes were assessed as six concepts for beam design in [5], and different types of impact drivers were considered as different concepts for the design of impact driver in [6].

In this study, we are mainly concerned with *multi-objective* MCO problems. Schematically, the target of solving multi-objective MCO problems can be understood from Fig. 1, where the objective space is shown for five potential concepts (C1-C5), along with their individual PFs. The aim of the MCO problem is not to achieve the approximation of these PFs individually, as shown in Fig. 1a, but instead to achieve the overall PF approximation across multiple concepts, as shown in Fig 1b. This overall PF approximation is referred to as s-Pareto in [7], [8] and C-Pareto in [9]. It can be seen that only parts of the individual PFs of some concepts (e.g. C1-C4) may appear in the overall PF, and some concepts (C5) may not contribute to the overall PF at all.

While the importance of considering multiple concepts has been discussed in the design-related literature, there currently exist relatively few dedicated and efficient algorithms to solve MCO problems. One of the plausible reasons contributing to this research gap is the unavailability of extensive benchmark problems in the MCO domain. In the evolutionary computation literature, the development of specific classes of algorithms



(a) (b)

Fig. 1. (a) PF of individual concepts and (b) Overall PF for MCO problem.

has often been driven by the design of the corresponding type of test problems [10], [11]. Such problems, by design, have known features (including true optimum) and span a range of mathematical/computational challenges, which assists in a comprehensive quantitative benchmarking of the algorithms in the field [10]. In some instances, even if the problems do not exhibit real-world features, they are still useful in identifying specific strengths and weaknesses of an algorithm. This, in turn, leads to the design of improved algorithms to address those aspects and, subsequently, also show improved performance for practical problems.

In this paper, we mainly intend to highlight the above observation in regard to multi-objective MCO by conducting a brief review and preliminary analysis of some existing test problems in the domain. We present a collection of test problems that have been used in the limited studies available on the topic in Section II, indicating some of their basic characteristics. Thereafter, in Section III, we list some of the limitations of these test problems through observation and/or preliminary experiments. We hope that this will help in understanding the scope for improvement and the systematic creation of more realistic and challenging problem sets in the future. Concluding remarks and potential future works are given in Section IV.

II. Review of Test Problems

As indicated previously, there is relatively scarce research in the field of MCO to date. Given that the dedicated MCO formulation itself is not so widely familiar to the researchers, the limited works in the domain have typically focused on illustrating the ideas through simple problems rather than constructing challenging instances of the problem. Some of these examples appear in works such as [8], [12]–[17]. Generally speaking, each problem is individually created to demonstrate certain scenarios rather than following a specific methodology to create a suite of instances. We present a brief discussion of a collection of such problems in this section. Given that the problems were not specifically named in the cited references, we list them as a series of problems for easy reference. The key attributes of these problems are listed in Table I next, followed by their discussion. The mathematical problem definitions can be found in the cited resources.

TABLE I
Existing MCO Test Problems

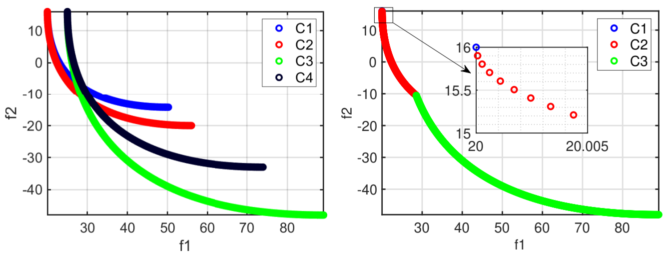
Name of Problem	No. of Concepts	No. of Objectives	No. of Variables	Type of Variables	No. of Constraints
Problem 1 [12]	4	2	1	Continuous	0
Problem 2 [14]	2	2	1, 2	Continuous	0
Problem 3 [14]	2	2	2	Continuous	2
Problem 4 [16]	2	3	3	Continuous	1
Problem 5 [15]	2	2	1	Continuous	0
Problem 5a [13]	2	2	1	Continuous	0
Problem 6 [15]	8	2	2	Continuous	2
Problem 7 [8]	2	2	1	Continuous	2
Problem 8 [13]	4	2	1	Continuous	0
Problem 9 [13]	8	2	1	Continuous	0
Problem 10 [17]	3	2	2, 3, 4	Continuous	0
Problem 11 [18]	2	2	1, 2	Continuous	0
Problem 12 [18]	2	2	3, 1	Continuous	0
Problem 13 [18]	2	2	2	Continuous	2
Problem 14 [18]	2	2	2	Continuous	2

Note that the PF shown subsequently for all test problems are approximations obtained through multi-objective search rather than theoretical sampling. To generate them, 31 independent runs of NSGA-II [19] on each concept were conducted with a population of 100 individuals, evolved for 100 generations. The SBX crossover and polynomial mutation were applied with a probability of 0.95 and 0.1, respectively, and distribution indices of 15 and 20, respectively. The solutions from the 31 runs for each concept were combined, and a nondominated (ND) sorting process was applied to obtain the PF approximation for each concept. Then, for generating overall PF, these ND solutions of each concept were combined. Nondominated sorting and distance-based subset selection [20] were then applied to the resulting set to pick 1000 final solutions that represent the overall PF.

a) Problem 1 [12]: This is an unconstrained one-variable bi-objective problem with four concepts. The objective functions of each concept are quadratic, with differences among them a resultant of different parameter values of the equations. The PFs of each concept are convex-shaped, and the overall PF is continuous (Fig. 2). There is a disparity in the magnitudes of the two objectives in the PF, which could create challenges for some algorithms that do not have normalization/scaling mechanisms. Note that for brevity, a concept is abbreviated by the letter ‘C’ for brevity in the following figures and discussion.

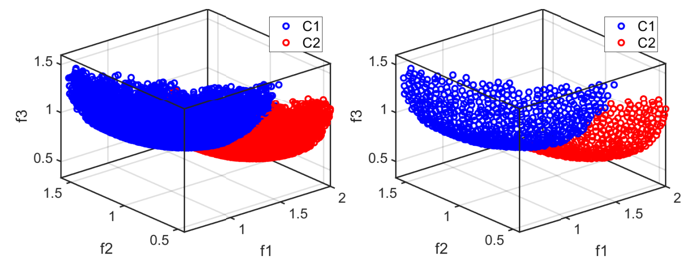
b) Problem 2 [14, Ex.1]: This is an unconstrained bi-objective optimization problem with two concepts. The first objective is linear for both concepts. In C1, the second objective function is quadratic, while in C2, it is a rational function. C1 yields a convex-shaped continuous PF with a similar range for each objective. On the other hand, C2 converges to a single solution, indicating nonconflicting objectives.

c) Problem 3 [14, Ex. 2]: The problem shown here is a slightly modified version of the one in [14]. To approximate the PF, with the ranges as shown and discussed in [14], the



(a) (b)

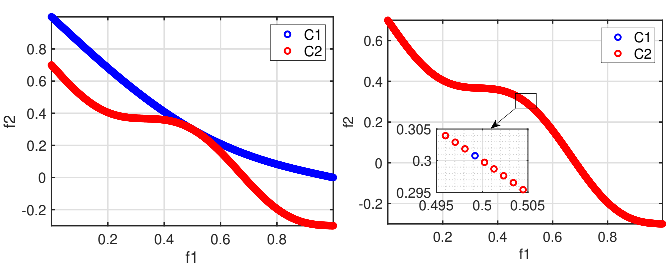
Fig. 2. (a) PF of each concept and (b) overall PF of Problem 1.



(a) (b)

Fig. 4. (a) PF of each concept and (b) overall PF of Problem 4.

ranges of the objectives need to be restricted, which we have done by introducing simple bound constraints on objectives in the modified version[1]. The constrained version is a bi-objective problem with two variables in the same domain. The first objective is a linear function, while the second one is a nonlinear function involving quadratic, linear and sinusoidal terms. C1's PF is convex-shaped, while C2's PF exhibits a mixed shape with convex and nonconvex regions. Both PFs are continuous. The overall PF (Fig. 3b) is nearly identical to that of C2, but a small portion of C1 is also included.

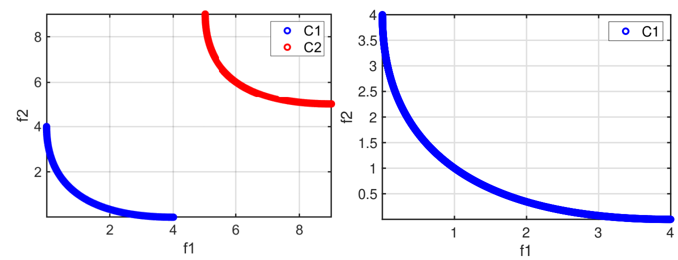


(a) (b)

Fig. 3. (a) PF of each concept and (b) overall PF of Problem 3.

d) Problem 4 [16, Ch.7, Ex. 7.3]: Unlike other problems, this one has three objectives in addition to constraints. It involves two concepts, each with three variables. Both concepts share the same set of linear objective functions, but they differ in their constraints. The feasible regions of each concept are spherical in shape. The PFs of each concept (Fig. 4a) are convex and continuous. The overall PF (Fig. 4b) comprises a continuous surface formed by combining parts of the fronts of both concepts. The trade-off ranges in each objective of each concept's PF and the overall PF lie are similar in magnitude. The domains of decision variables for each concept are also the same.

e) Problem 5 [15, Ex. A]: Problem 5 is an unconstrained single-variable bi-objective optimization problem with two concepts. The objective functions of the concepts are quadratic, with a different set of constant terms between the concepts. Both concepts' PFs (Fig. 5a) and the overall PF (Fig. 5b) are convex-shaped and continuous. C1 dominates C2, making the overall PF identical to C1's PF. The trade-off range magnitudes in each objective of each concept's PF are similar.

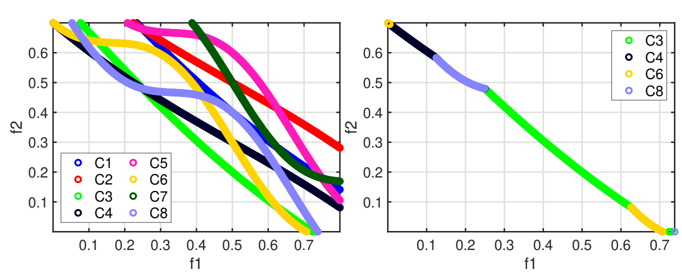


(a) (b)

Fig. 5. (a) PF of each concept and (b) overall PF of Problem 5.

f) Problem 5a [13, Ex. 1]: This extended version of Problem 5 includes a third concept with different parameter values and a decision variable domain of [-10,10] instead of [-5,5]. C3's PF shape matches C1 and C2 from Problem 5. Similar to the original version, C1 dominates C3. Thus, the overall PF resembles that of the original problem.

g) Problem 6 [15, Ex. C]: This is a bi-objective optimization problem having eight concepts with two variables in the same domain. Like Problem 3, the first objective function is a single-variable linear function, while the second objective function is a nonlinear two-variable function involving quadratic, linear, and sinusoidal terms. The PFs of all concepts (Fig. 6a) mixed (convex/concave) shapes. The overall PF (Fig. 6b) consists of segments from four of the concepts.



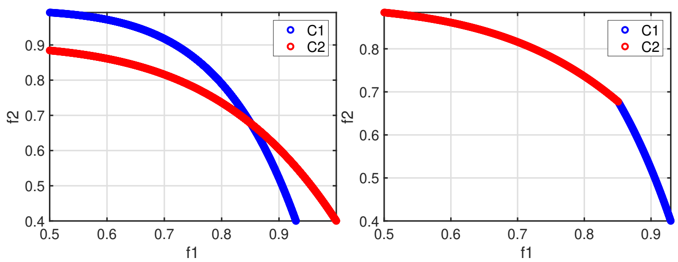
(a) (b)

Fig. 6. (a) PF of each concept and (b) overall PF of Problem 6.

h) Problem 7 [8, Ex. 1]: It is a two-concept problem where the first objective is sinusoidal for both concepts.

[1]Similar modifications are also done for Problems 6, 7, 13 and 14

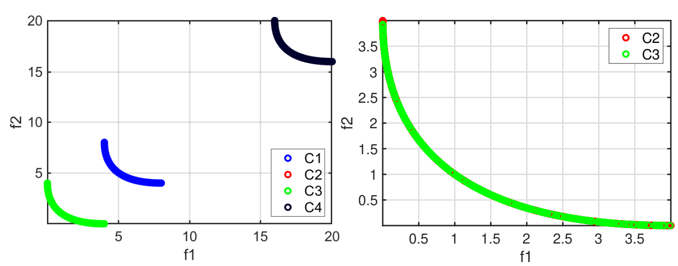
However, the second objective function differs between the two concepts, though they both involve power transformations of the sine function. The variable range was not explicitly mentioned in [8]. Observing the nature of the objective functions, we have assumed the variable range to be $[0, \pi/2]$. Moreover, [8] examines this problem in a stochastic environment; we present it here in a deterministic context. The PFs of each concept (Fig. 7a) are concave-shaped and continuous. The overall PF (Fig. 7b) is also concave-shaped, resulting from the contribution of both concepts' fronts. The trade-off range magnitudes in each objective of C1 and C2's PFs are dissimilar.



(a) (b)

Fig. 7. (a) PF of each concept and (b) overall PF of Problem 7.

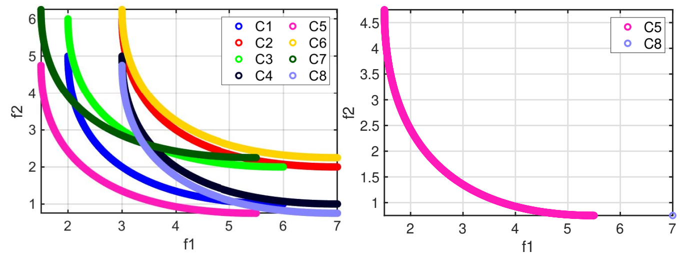
i) Problem 8 [13, Ex. 2]: Problem 8 is an unconstrained single-variable bi-objective optimization problem with four concepts. Each concept contains different variations of quadratic objective functions and has convex-shaped, continuous PFs. The problem was designed in [13] to evaluate algorithm performance when two concepts share the same front. Upon setting specific parameter values for C2 and C3, their objective functions become identical. Both C2 and C3 dominate the other concepts. Consequently, the overall PF (Fig. 8b) is identical to the PFs of C2 and C3. The trade-off range magnitudes in each objective of each concept's PF and the overall PF are similar.



(a) (b)

Fig. 8. (a) PF of each concept and (b) overall PF of Problem 8.

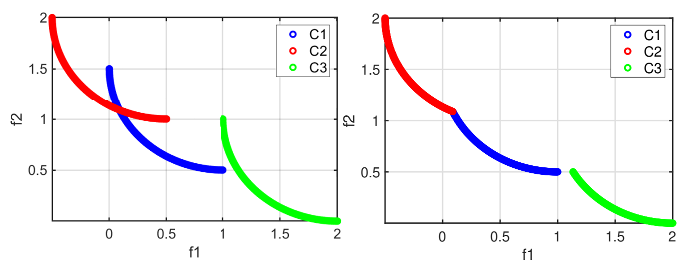
j) Problem 9 [13, Ex. 5]: Problem 9 is a single-variable unconstrained bi-objective optimization problem with eight distinct concepts. Similar to Problems 1, 5, and 8, the objectives of each concept represent different variations of a quadratic function, resulting in convex-shaped, continuous Pareto fronts (Fig. 9a). However, unlike the other mentioned problems, the overall PF is disconnected (Fig. 9b). C5 dominates all other concepts except for a small portion of C8. Consequently, the overall PF is mainly the same as that of C5, with an additional small segment contributed by C8.



(a) (b)

Fig. 9. (a) PF of each concept and (b) overall PF of Problem 9.

k) Problem 10 [17, Appendix H, Ex. 1]: Problem 10 is an unconstrained bi-objective optimization problem with three concepts. Unlike most of the previous problems, the number of decision variables differs for each concept, but their domains are the same. Each concept involves different combinations of objective functions, including trigonometric, linear, and constant terms. All concepts have convex-shaped and continuous PFs (Fig. 10a). The overall PF is formed by contributions from all three concepts and has two disconnected segments (Fig. 10b). While the magnitudes of trade-off ranges in each objective of each concept's PF are similar, the overall PF's trade-off ranges are marginally dissimilar.

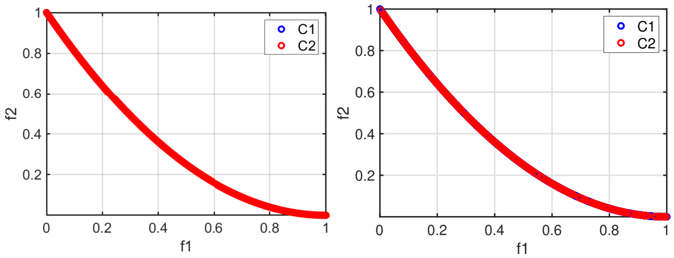


(a) (b)

Fig. 10. (a) PF of each concept and (b) overall PF of Problem 10.

l) Problem 11 [18, Ch. 4, Ex. 4.1.1-B]: Problem 11 is an unconstrained bi-objective optimization problem with two concepts. The number of decision variables differ for each concept, but their domains are the same. For both concepts, the first objective function is linear, while the second objective function is quadratic. Both concepts have convex-shaped continuous PF (Fig. 11a). The overall PF (Fig. 11b) is identical to the PFs of C1 and C2. The trade-off range magnitudes in each objective of each concept's PF and the overall PF are similar.

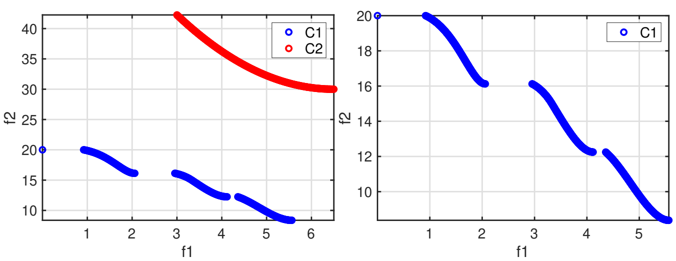
m) Problem 12 [18, Ch. 4, Ex. 4.1.2-E]: This is an unconstrained bi-objective optimization problem with two concepts. C1 is a modified version of the KUR [21] MOP test problem, with both the original objective functions shifted



(a) (b)

Fig. 11. (a) PF of each concept and (b) overall PF of Problem 11.

by 20 units. KUR is scalable in terms of the number of decision variables; 3 variables are considered in C1. In C2, the first objective is a linear function, whereas the second one is quadratic. C1's PF is disconnected, and C2's is convex and continuous (Fig. 12a). C1 dominates C2, making the overall PF (Fig. 12b) identical to C1's PF. The trade-off range magnitudes in each objective of each concept's PF are dissimilar.

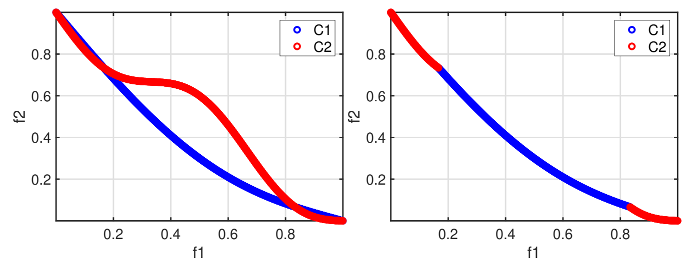


(a) (b)

Fig. 12. (a) PF of each concept and (b) overall PF of Problem 12.

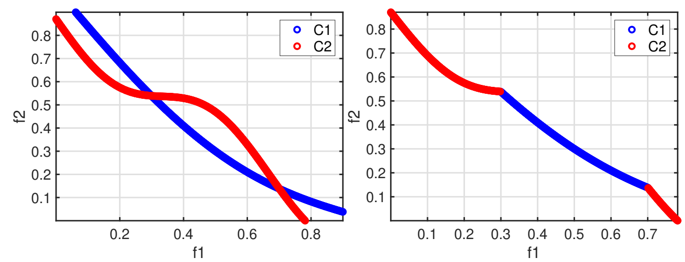
n) Problem 13 [18, Ch. 4, Ex. 4.1.1-A]: Apart from the modifications brought by introducing bound constraints as discussed in Problem 3, we have also slightly modified the second objective function of C2. In the original version, as stated in [18], the constant term was 0.75. However, to match the PF, we have changed the constant term to 1. The modified version is a bi-objective problem with two variables in the same domain. The first objective is a linear function, while the second is a nonlinear function involving quadratic, linear and sinusoidal terms. C1's PF is convex-shaped, while C2's PF exhibits a mixed shape with convex and nonconvex regions (Fig. 13a). Each concept substantially contributes to the overall PF (Fig. 13b).

o) Problem 14 [18, Ch. 4, Ex. 4.1.2-A]: It is a bi-objective problem with two variables in the same domain. The first objective is a linear function, while the second one is a nonlinear function involving quadratic, linear and sinusoidal terms. C1's PF is convex-shaped, while C2's PF exhibits a mixed shape, combining convex and nonconvex regions (Fig. 14a). Both PFs are continuous. Each concept substantially contributes to form the overall PF (Fig. 14b).



(a) (b)

Fig. 13. (a) PF of each concept and (b) overall PF of Problem 13.



(a) (b)

Fig. 14. (a) PF of each concept and (b) overall PF of Problem 14.

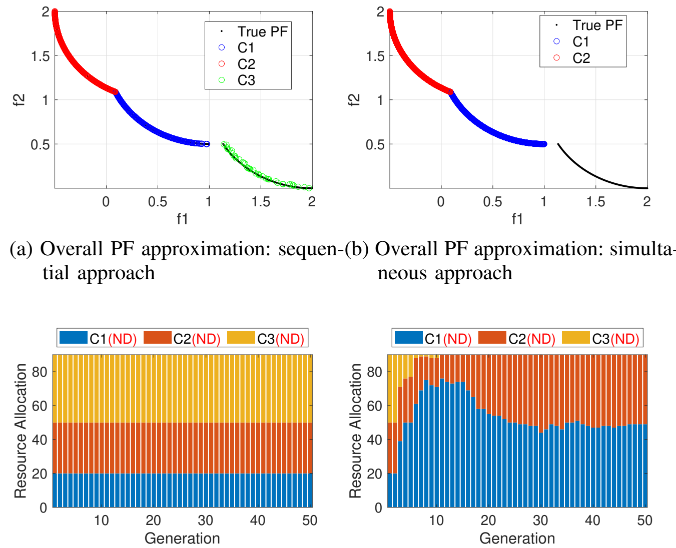
III. Preliminary Observations and Limitations of Test Problems

To gain further insights into the problems, we conducted preliminary investigations using Strategies 1-2 from [3]. Strategy 1 essentially conducts a search for each concept individually, whereas Strategy 2 conducts a simultaneous search with a computational budget (function evaluations) allocated to the concepts based on their performance. Strategy 1 is also referred to in the literature as the *sequential* approach. For Strategy 2, we set the minimum guaranteed evaluation for each concept (NM_i) to 0 instead of 50% in [3] of the initial population size to observe the strategy's performance under purely performance-based computing resource allocation. The maximum number of function evaluations (FE_{max}) is set at 50 times the sum of the initial populations of all concepts, and the initial population size N_i for each concept is set at 10 times the number of decision variables D_i. The other parameters used for the experiments remained identical to those stated in [3]. We ran each strategy 31 times on these problems. We excluded Problem 2 in the experiments since it has nonconflicting objectives for C2.

The problem formulations indicate that these problems are relatively straightforward, with a small number of decision variables (usually 1-2). Additionally, constrained problems typically involve 1-2 constraints that are not highly nonlinear. The applied strategies were able to find feasible solutions from the initial population for all runs of all the constrained problems. It is also observed that their PFs lack diversity and controllability in terms of shapes, and irregularities such as disconnected patches or a mix of different PF shapes (e.g., convex, concave, linear) are uncommon. Moreover, the scalability of the problems in terms of the number of concepts, objective functions, decision variables and constraints has not been inherently considered in the problem formulations. Thus, while it may be possible to extend them in theory, it would require constructing them manually for each instance. Most of the example problems with > 1 design variables have variable domains of identical magnitude, contrary to the recommendation for dissimilar magnitudes [11]. Likewise, the magnitudes of trade-off ranges in each objective of existing test problems are often similar, whereas dissimilarity is recommended [11] to better simulate real-world scenarios.

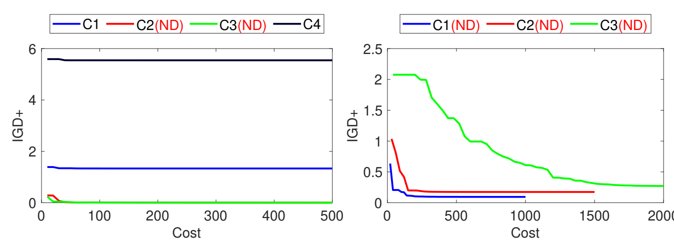
Some previous studies [3], [14] have reported an interesting phenomenon in MCO problems where some concepts initially perform poorly compared to others in the early search stages. However, with sufficient computational resources, these initially inferior concepts eventually contribute solutions to the PF. If the allocation of computational resources is solely driven by performance, solutions from these initially inferior concepts might be prematurely eliminated due to the selection pressure early in the search. This premature elimination of concept phenomena is observed in Problems 3 (C1), 9 (C8), 10 (C3) and 11 (C2) among the discussed test problems. Fig. 15 illustrates this effect by displaying the overall PF approximations and the concept-wise resource allocation plots of Problem 10 for both the sequential (Strategy 1) and simultaneous (Strategy 2) approaches. These plots are associated with the runs that achieved the median IGD$^+$ value [22]. In the simultaneous approach, where resource allocation is performance-driven, initially under-performing C3 receives fewer evaluations and is eliminated from the search process early (Fig. 15d), resulting in no contribution to the overall PF approximation achieved (Fig. 15b). Conversely, the sequential approach (Strategy 1) allocates resources to each concept regardless of its performance (Fig. 15c), allowing C3 to contribute to the overall PF (Fig. 15a). To note, ND after concept ID highlights the concepts that contribute to the overall PF of the problem.

In most of the discussed problems, the participating concepts exhibit similar levels of individual difficulty in terms of convergence. The similarity in difficulty among the concepts arises from the similar nature of objective functions and/or constraints (where applicable) in the participating concepts, along with an equal number of decision variables in each concept. Fig. 16a illustrates that the convergence rates, with respect to the overall PF and in terms of IGD$^+$, are approximately the same for each concept of Problem 8 (noting that the ND concepts converge to a lower IGD$^+$ values than non-ND concepts). In contrast, the relative difficulty among the concepts in Problem 10 varies due to differences in the number of decision variables. Intuitively, concepts with a higher number of variables are expected to encounter greater convergence difficulty. Fig. 16b confirms the intuition, showing that the convergence rate of each concept differs and follows a descending order with an increase in the number of decision variables for the concepts. Note that the plots in Fig. 16 correspond to runs that achieved a median IGD$^+$ value for the sequential approach on Problems 8 and 10. The codes and data used for the above analyses can be obtained from the first author for research purposes.



(a) Overall PF approximation: sequential approach
(b) Overall PF approximation: simultaneous approach
(c) Concept-wise resource allocation: sequential approach
(d) Concept-wise resource allocation: simultaneous approach

Fig. 15. Premature elimination of nondominated C3 in Problem 10: comparing overall PF (top) and concept-wise resource allocation (bottom) between sequential (left) and simultaneous (right) approaches.



(a)
(b)

Fig. 16. (a) Similar level of convergence difficulty among concepts of Problem 8; (b) mixed level of convergence difficulty among concepts of Problem 10.

From the discussion above, it can be understood that in the current form, the test problems lack sufficient challenges in the search landscape, as well as the ability to easily scale or control the difficulty of the problems. Thus, while they may serve as good illustrations, they may not provide realistic challenges for the development of advanced MCO algorithms, highlighting the need for further research in MCO problem construction.

Lastly, it is worth noting that in addition to the test problems discussed above, a number of practical case studies have also been modelled in MCO format. Real-world MCO problems often involve complex simulations, physical experiments, or computationally intensive models to evaluate the objective functions and/or constraints. For example, the objective functions of lattice structure [3], rigidified inflatable structure [16], commuter aircraft [23] etc. problems require computationally intensive simulations. These evaluations can be time-consuming and resource-intensive, limiting the number of function evaluations that an optimization algorithm can perform. Moreover, many real-world MCO problems exhibit highly nonlinear and black-box constraint functions (e.g., rigidified inflatable structure [16]). They may also involve equality constraints (e.g., aegis UAV problem [24]) that make the problems particularly challenging for metaheuristic methods. These features form additional considerations for the design of more practical test functions for benchmarking MCO algorithms.

IV. Conclusion and Future Research

The primary objective of this study is to review some existing test problems in the multi-objective MCO field and highlight some of their limitations. For the future developments of advanced MCO algorithms, these research gaps could be targeted in order to develop more diverse and challenging test problem suites. A potential approach to develop MCO test problems, as presented in [25]–[28], can be utilizing problems from existing test suites as concepts. Each problem from the existing test suites may represent a distinct concept characterized by its objective functions, constraints, and decision variables. By combining multiple such concepts, we can generate more practical MCO problems and incorporate specific challenges as needed. The authors are currently working along these lines to come up with a benchmark generator and specific instances that span a range of practical and theoretical challenges. In addition, the development of more advanced algorithms will also be investigated in future works.

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