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Hand Inertial Parameters Calculation for any Position Through the Kinematic Model

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***Abstract*—In biomechanics, the calculation of inertial parameters for the upper and lower limbs is studied for motion analysis or the design of prostheses or exoskeletons. However, the calculation of inertial parameters for the hand is performed without considering that the geometry of this segment can change depending on the posture. This work presents a geometric method based on the kinematic model to estimate the inertial parameters of the hand segment for different hand postures. The resulting inertia tensor is calculated at the center of mass according to the segment axes' International Society of Biomechanics (ISB) designation. It considers the principal moments of inertia and the products of inertia of the hand segment. To demonstrate the use of this tool, six healthy subjects participated. The anthropometric measurements of their hand were obtained, the inertial parameters were calculated with our proposal, and they were compared with two methods, Dumas and De Leva, using the Euclidean and Frobenius norms for the center of mass and the inertia tensor, respectively. The mean difference and SD between the proposed method for the relaxed hand position against the Dumas method is 0.0049 m (SD 0.002) and 0.00016 $\cdot 10^{-3} kg-m^2$ (SD 0.00009) and the De Leva method is 0.011 m (SD 0.0013) and 0.00023 $\cdot 10^{-3} kg-m^2$ (SD 0.00004) for the center of mass and the inertia tensor, respectively. However, our method can be extended to different hand positions. The proposed method can be used in applications such as the analysis of the three-dimensional motion of the upper limb or in the design of biomedical devices such as hand or wrist and forearm exoskeletons.**

***Index Terms*—Hand segment inertial parameters; Hand kinematic model; Hand biomechanics.**

I. Introduction

Dynamic analysis plays a key role in understanding human motion, modeling sports performance, and designing biomedical devices such as prostheses and exoskeletons. The simulation of human motion helps to understand motor diseases or pathological conditions and facilitates the proposal of improved clinical metrics and indices. Consequently, there is a growing interest in developing more realistic models, leading to the adoption of dynamic models of the human body. For example, when simulating limb movements in patients with conditions such as muscular dystrophy, spasticity, or Parkinson's disease, factors such as stiffness and elasticity become critical considerations [1–3].

Kinematic and dynamic models are essential to simulate a limb moving under specific forces and moments. A kinematic model is typically computed using an orthonormal segment coordinate system (SCS) with methods such as Euler angles, homogeneous matrices, or braces and quaternions [4]. On the other hand, a dynamic model requires several parameters such as gravity, external forces, and intersegmental forces and moments [5]. The calculation of intersegmental forces depends on the knowledge of the inertial parameters of each body segment, such as the mass, the position of the center of mass and the inertia tensor [6].

Traditionally, the estimation of inertial parameters for human body segments is achieved using regression models [7–9] based on data obtained from human cadaver studies [10–13]. In addition, some methods approximate the inertial parameters for segments such as legs or arms [14, 15]. However, for the hand segment, it is commonly assumed to have a fixed posture, neglecting variations in the inertial parameters with different hand postures. Although these variations may not have a significant impact on the overall analysis of human body movements, especially in gait analysis, they can lead to increased errors in the analysis of hand movements, which is particularly relevant in prosthetic and orthotic design where hand posture is critical for accuracy [16].

II. Methodology

A. Subjetcs

Six men between 20 and 28 aged participated voluntarily. They signed a consent to participate and followed the instructions. The anthropometric measurements of each segment of their right hands were obtained acording to Fig. 1.

B. Methods

To calculate the inertial parameters of the hand, the body segment is divided into 16 rigid elements, referred to as

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hand segments, as illustrated in Fig. 1. These segments are associated with the hand axis system, as shown in Fig. 3. The method involves localizing all hand segments during specific hand postures using the presented direct kinematic model. This model allows us to determine the attitude and position of each hand segment at its center of mass. Fig. 2 contains a visual representation of the steps to illustrate the general workflow of the proposed method.

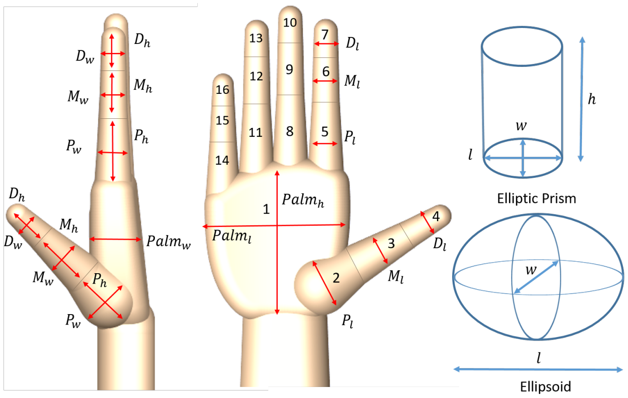

Fig. 1. Segmentation of the hand and anthropometric measurements taken on each subject

C. Segmentation

In our approach, we utilize standard geometries to approximate the inertial values of each hand segment, assuming a constant density. The mass of each hand segment is calculated as a percentage of the total hand mass, as presented in Table I. The calculation of the hand mass is based on a percentage of the total body weight, represented by Equation (1), acording to [14, 15].

$$M_h = 0.006 * B_m \tag{1}$$

where M_h is the mass of the hand and B_m is the total mass of the body. Table I shows the corresponding percentages of the total mass of the hand and the geometry applied to each element; these percentages were obtained by geometric weighting, necessary to approximate the inertial parameters of each hand segment. The geometric properties were determined based on the anthropometric measurements of each segment.

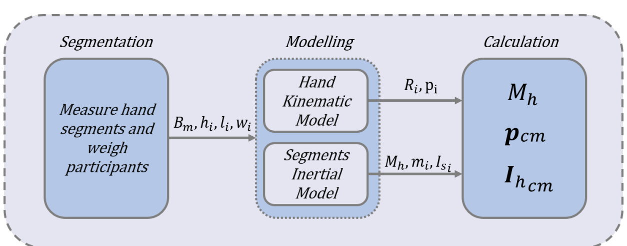

Fig. 2. General scheme of the method used to calculate the hand inertial parameters

TABLE I
HAND SEGMENT MASS DISTRIBUTION AND ASSIGNED GEOMETRY

Segment	Code	% of segment mass	Assigned geometry
Palm	1	$0.5 * M_h$	Ellipsoid
Thumb I	2	$0.1 * M_h$	Ellipsoid
Thumb II	3	$0.05 * M_h$	Elliptic prism
Thumb III	4	$0.03 * M_h$	Elliptic prism
Index I	5	$0.04 * M_h$	Elliptic prism
Index II	5	$0.025 * M_h$	Elliptic prism
Index III	7	$0.015 * M_h$	Elliptic prism
Middle I	8	$0.045 * M_h$	Elliptic prism
Middle II	9	$0.03 * M_h$	Elliptic prism
Middle III	10	$0.015 * M_h$	Elliptic prism
Ring I	11	$0.04 * M_h$	Elliptic prism
Ring II	12	$0.025 * M_h$	Elliptic prism
Ring III	13	$0.015 * M_h$	Elliptic prism
Little I	14	$0.035 * M_h$	Elliptic prism
Little II	15	$0.02 * M_h$	Elliptic prism
Little III	16	$0.015 * M_h$	Elliptic prism

D. Kinematic modeling

The hand's structure can be represented as a tree-like configuration, with the palm serving as the main body, and each finger is modeled as an open chain. The kinematic model employed to calculate the hand's inertial parameters is illustrated in Fig. 3 (a), and it focuses only on the actuated joints of the hand. For each finger, we have assigned three hand segments (proximal, middle, and distal phalanges), and the direct kinematic model for each segment is calculated at its mass center using the chain rule of homogeneous transformations [17].

Fig. 3 (b) displays the coordinate systems assigned to the index finger and thumb, with the local axis system positioned at the base of each segment. The homogeneous transformation for each axis system is determined using Equation 2. The direct kinematics for the 16 segments of the hand are calculated using Equation 3, which involves the multiplication of the homogeneous transformations of the palm and parent phalanges involved in the movement of the segment.

$$T^i_{i-1} = \begin{bmatrix} R^i_{i-1} e^{[a_i \times]\theta_i} & p_{i/i-1} \\ \vec{0} & 1 \end{bmatrix} \in SE(3) \subset \mathbb{R}^{4\times 4} \tag{2}$$

$$T^i_0 = \prod_{j=1}^{i} T^j_{j-1} = \begin{bmatrix} R_i & p_i \\ \vec{0} & 1 \end{bmatrix} \tag{3}$$

where T^i_{i-1} represents the homogeneous transformation matrix that denotes the orientation R^i_{i-1} and position vector $p_{i/i-1}$ with respect to the parent axis system. This transformation matrix is used to describe how the coordinate system of the segment "i" is related to its parent segment "i-1". The term $e^{[a_i \times]\theta_i}$ in Equation 2 represents the exponential expression of the rotation matrix using axis-angle representation. In this expression, a_i represents the joint axis of the segment "i" and θ_i is the angle of rotation of the segment. The cross-product notation $[a_i \times]$ represents the skew-symmetric matrix associated with the joint axis a_i.

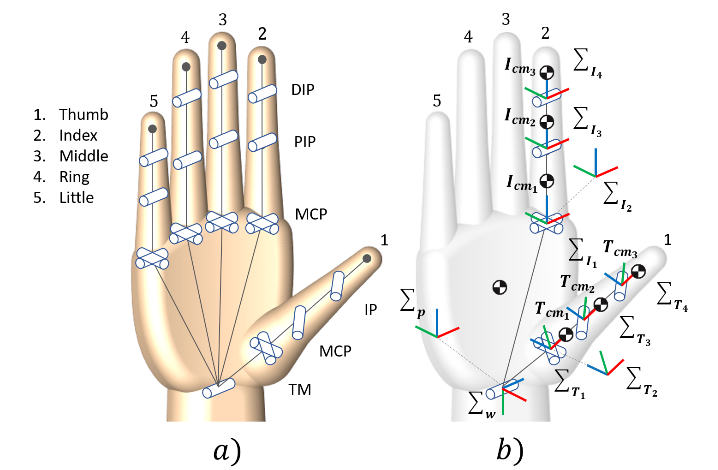

Fig. 3. a) 21 DOF hand kinematic model used to calculate the hand inertial parameters b) Assignments of the joint axes systems for the index finger and thumb

Fig. 3 (b) illustrates the local axes assignment for the thumb finger, which is the most complex finger due to its extensive range of movements and its position with respect to the palm. To accurately model the kinematics of the thumb finger, non-orthogonal methods are utilized, considering the intricate movements it can achieve.

E. Calculating the center of mass

To calculate the centre of mass, we use Equation (4).

$$p_{cm} = \frac{\sum_{i=1}^{16} m_i p_i}{M_h} \tag{4}$$

where m_i is the mass of each hand segment obtained as shown in Table I, p_i is the position of the centre of mass at the hand local axis, and M_h is the total hand mass.

F. Calculating inertia tensor

The hand inertia tensor is calculated as the sum of total inertial contributions of each individual segment at the hand local axis, according to the hand posture, see Equation (5).

$$I_h = \sum_{i=1}^{16} I_{s_i} \tag{5}$$

where I_{s_i} is the inertia tensor of segment i and I_h is the hand inertia tensor, both calculated on the hand local axis.

To calculate each segment inertia tensor we use an adequation of the 3D parallel axis theorem, see Equation (6).

$$I_{s_i} = R_i I_i R_i^T - m_i [p_i \times]^2 \tag{6}$$

where R_i and p_i are the rotation matrix and the position vector respectively of segment i at the hand local axis, extracted from the direct kinematics of each segment. I_i is the segment inertia tensor at its centre of mass, calculated according to the assigned geometry in Table I. The use of R_i in $R_i I_i R_i^T$ is due to the similarity transformation to change the segment inertia tensor from the segment local axis to the hand local axis. Notice that R_i and p_i are dependent on the hand configuration posture. $[p_i \times]$ is a 3×3 skew-symmetric matrix called cross product operator (CPO), [18]. To calculate the inertia tensor at the hand centre of mass we use the Equation (7).

$$I_{cm} = I_h + M_h [p_{cm} \times]^2 = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix} \tag{7}$$

where M_h is the hand segment mass calculated in (1), p_{cm} is the center of mass calculated in (4) and I_h is the hand inertia tensor calculated in (5).

III. Results

Table II presents a concrete example of the inertial parameters calculated with the proposed method. This example corresponds to a single participant, referred to as Subject 1, and covers four different hand configurations. These configurations are the extended hand (ST), the relaxed hand (REL), full fist (FF) and the ring gesture (RN), as shown in the Fig. 4. The mass attributed to the hand, according to the model visualized in Fig. 4, is quantified as 0.570 kg. This quantification comes from calculations for a 27 year old male participant with a body mass of 95 kg. It is important to emphasize that all inertia tensor matrices shown in the table II were calculated with respect to the single center of mass corresponding to each individual hand pose. The Euclidean norm differences between the centers of mass of ST, RN, and FF compared to the relaxed hand are 0.005, 0.0169, and 0.023 meters, respectively. In addition, the differences in inertia tensors calculated using the Frobenius rule in ST, RN, and FF relative to REL are 0.000179, 0.000178, and 0.000977 kg-cm², respectively.

TABLE II
EXAMPLE OF INERTIAL PARAMETERS, CENTER OF MASS AND INERTIA TENSOR, OBTAINED WITH THE PROPOSED METHOD FOR ONE SUBJECT AND FOUR HAND POSITIONS

Hand Inertial Parameters		
Hand Position	**Center of mass** (m)	**Inertia tensor** ($kg \cdot m^2 \cdot 10^{-3}$)
ST	$\begin{pmatrix} 0.0018 \\ -0.0720 \\ 0.0085 \end{pmatrix}$	$\begin{pmatrix} 1.231 & -0.011 & -0.085 \\ -0.011 & 0.460 & -0.156 \\ -0.085 & -0.156 & 1.131 \end{pmatrix}$
REL	$\begin{pmatrix} 0.0066 \\ -0.0709 \\ 0.0078 \end{pmatrix}$	$\begin{pmatrix} 1.256 & 0.124 & -0.118 \\ 0.124 & 0.592 & -0.164 \\ -0.118 & -0.164 & 1.247 \end{pmatrix}$
FF	$\begin{pmatrix} -0.0107 \\ -0.0567 \\ 0.0031 \end{pmatrix}$	$\begin{pmatrix} 0.467 & 0.040 & -0.051 \\ 0.040 & 0.445 & -0.035 \\ -0.051 & -0.035 & 0.687 \end{pmatrix}$
RN	$\begin{pmatrix} -0.0077 \\ -0.0624 \\ 0.0049 \end{pmatrix}$	$\begin{pmatrix} 0.982 & -0.102 & -0.185 \\ -0.102 & 0.658 & -0.167 \\ -0.185 & -0.167 & 1.246 \end{pmatrix}$

Additionally, Table III provides an overview of the aggregated results obtained from all participating subjects for the relaxed hand posture. It gathers a comparative analysis of the proposed approach (PM) with respect to the hand inertia parameter estimates obtained using the De Leva method (DLM) and the Dumas method (DM), as explained in ŠciteDumas2018. This evaluation has been meticulously synthesized in Table IV, using the Euclidean norm for the evaluation of the centers of mass and the Frobenius norm for the evaluation of the inertia tensors.

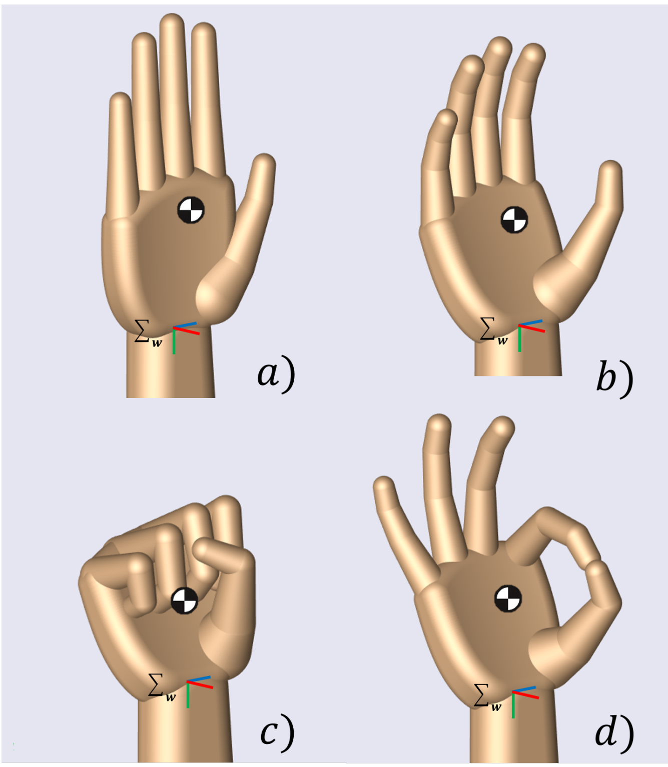

Fig. 4. Proposed hand positions. a) extended palm or straight (ST), b) relax (REL), c) full fist (FF), and d) ring (RN), segment axes according to ISB.

IV. Discussion

In this study, we compared the widely used De Leva method (DLM) with the more recent Dumas method (DM) for calculating the inertial parameters of the hand. DLM provides values on the main diagonal of the inertia tensor, assuming zero for the products of inertia. At the same time, DM considers the products of inertia, making it more accurate for the non-symmetric geometry of the hand. However, both methods calculate the inertia tensor for a single position, which is not reported, limiting their ability to capture the hand's variations during different movements.

Our proposed method calculated the inertial parameters for four different hand positions to address this limitation. The results for the relaxed hand posture were comparable to those obtained by DM and DLM, indicating that these methods effectively estimate the parameters for a relaxed hand posture. However, the advantage of our proposed method lies in its ability to provide inertial parameters for multiple hand positions, making it more versatile and applicable in different scenarios.

The results obtained from our method have valuable implications for various applications. In designing exoskeletons or prosthetic devices for the hand, accurate estimation of inertial parameters is critical for achieving optimal performance and functionality. Our method's ability to capture the inertial variations of the hand during different postures ensures that the designed devices can adapt to different tasks and activities, improving user experience and efficiency.

Furthermore, in the field of dynamic analysis of the hand, understanding the hand's motion and its inertial properties is essential for biomechanical research and clinical applications. The ability of our proposed method to provide detailed and accurate inertial parameters for different hand positions allows researchers and clinicians to gain insight into hand movements during different tasks and assess potential impacts on performance or rehabilitation outcomes.

V. Conclusions

The presented method allows to compute the inertial parameters of the hand segment at different positions, which we compared with existing methods that compute only one hand position. This result has many applications, such as the design of exoskeletons or the dynamic analysis of hand motion.

For future work, we plan to apply computer vision techniques to the measurement of the hand segments, which will facilitate the computation of the kinematic model and the measurement of the hand segments.

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TABLE III

COMPARISION OF INERTIAL PARAMETERS OBTAINED WITH DUMAS, DELEVA AND OUR PROPOSAL FOR THE 6 SUBJET HANDS IN RELAX POSITION

Subjet	Method	Mass (kg)	**Center of mass** (m)			**Inertia tensor at center of mass** ($kg-m^2\cdot 10^{-3}$)					
		m	p_{cm_x}	p_{cm_y}	p_{cm_z}	I_{xx}	I_{yy}	I_{zz}	I_{xy}	I_{xz}	I_{yz}
1	PM	0.5700	0.0066	-0.0709	0.0078	1.2566	0.5924	1.2477	0.1249	-0.1186	-0.1647
	DM	0.5700	0.0069	-0.0705	0.0063	1.4966	0.5808	1.2613	0.1947	0.0905	-0.1609
	DLM	0.5795	0	-0.0659	0	1.0620	0.6489	1.5914	0	0	0
2	PM	0.4500	0.0065	-0.0855	0.0027	1.2488	0.4869	1.3209	0.2113	-0.0484	-0.1728
	DM	0.4500	0.0077	-0.0793	0.0071	1.4953	0.5803	1.2602	0.1945	0.0904	-0.1607
	DLM	0.4575	0	-0.0732	0	1.0350	0.6324	1.5511	0	0	0
3	PM	0.5580	0.0080	-0.0725	0.0071	1.2989	0.6165	1.3157	0.1910	-0.1071	-0.1706
	DM	0.5580	0.0069	-0.0705	0.0063	1.4650	0.5685	1.2347	0.1906	0.0886	-0.1575
	DLM	0.5673	0	-0.0659	0	1.0396	0.6352	1.5579	0	0	0
4	PM	0.6300	0.0070	-0.0777	0.0069	1.6079	0.7525	1.6815	0.2310	-0.1313	-0.1777
	DM	0.6300	0.0069	-0.0705	0.0063	1.6541	0.6419	1.3940	0.2152	0.1000	-0.1778
	DLM	0.6405	0	-0.0659	0	1.1737	0.7172	1.7590	0	0	0
5	PM	0.4920	0.0047	-0.0733	0.0039	1.0455	0.4213	1.1057	0.1740	-0.0445	-0.0939
	DM	0.4920	0.0060	-0.0617	0.0055	0.9890	0.3838	0.8335	0.1286	0.0598	-0.1063
	DLM	0.5002	0	-0.0659	0	0.9166	0.5601	1.3737	0	0	0
6	PM	0.6900	0.0082	-0.0732	0.0084	1.6605	0.7745	1.6490	0.2035	-0.1537	-0.2074
	DM	0.6900	0.0069	-0.0705	0.0063	1.8116	0.7030	1.5268	0.2356	0.1095	-0.1947
	DLM	0.7015	0	-0.0659	0	1.2855	0.7855	1.9265	0	0	0

TABLE IV

COMPARISION OF CENTER OF MASS (p_{cm}) AND INERTIA TENSOR (I_{cm}) OBTAINED WITH DUMAS, DELEVA AND OUR PROPOSAL FOR THE HAND IN RELAX POSITION

Subjet	p_{cm} **diference (m)**		I_{cm} **diference** ($kg-m^2\cdot 10^{-3}$)		**Body Data**		
	DM	DLM	DM	DLM	Weigh (kg)	Height (m)	BMI
1	0.0031	0.0102	0.0003	0.0002	95	1.78	30.0
2	0.0068	0.0121	0.00004	0.00029	75	1.82	22.6
3	0.0075	0.0143	0.00015	0.00028	93	1.76	30.0
4	0.0022	0.0110	0.00021	0.00024	105	1.8	32.4
5	0.0064	0.0109	0.00005	0.00024	82	1.72	27.7
6	0.0037	0.0128	0.00021	0.00016	115	1.75	37.6
Mean	0.00495	0.01188	0.00016	0.00023	94.16	1.77	30.1
SD	0.00202	0.00137	0.00009	0.00004	13.35	0.03	4.5

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