2023 IEEE Symposium Series on Computational Intelligence (SSCI)
Mexico City, Mexico. December 5-8, 2023

Multimodal Multi-objective Football Game Algorithm for Optimizing Test Task Scheduling Problems

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***Abstract*—This paper presents an innovative approach to addressing the complex challenges posed by Test Task Scheduling Problems (TTSPs) through the utilization of Multimodal Multi-objective Football Game Algorithm (MM-FGA). TTSPs hold significant importance in industries where testing plays a critical role in ensuring product quality. This study outlines the integration of MM-FGA with the Normalized Factor Random Key (NF-RK) encoding scheme, tailored to the discrete nature of TTSPs. Four real-world TTSPs are investigated with the objectives of minimizing makespan and mean workload. Comparative analyses are conducted against other prominent multiobjective algorithms, including NSGA-II, DN-NSGA-II, Tri-MOE-TA&R, MP-MMEA, and MO-Ring-PSO-CD. The results exhibit MM-FGA's competitive performance, particularly in terms of the diversity of solutions obtained in the decision space. This underscores MM-FGA's prowess in addressing the multimodality challenges of optimization problems. The study further suggests the prospect of advancing step-size control strategies through meta-optimization, aiming to refine the algorithm's exploration and exploitation balance for even more potent optimization outcomes. Overall, MM-FGA demonstrates promise in solving multimodal multiobjective discrete problems like TTSPs, while indicating room for future enhancements.**

Keywords—Optimization, Multimodal, Multi-objective, Test Task Scheduling Problem, Encoding Scheme

I. Introduction

Amidst a wide array of scheduling predicaments, Test Task Scheduling Problems (TTSPs) hold a distinct significance within industries where testing stands as a cornerstone in ensuring product quality and reliability. These problems surface in sectors like automotive, aerospace, and electronics, where exhaustive testing is of paramount importance. Effectively scheduling test tasks assumes criticality in curbing testing duration, steering workload distribution, and optimizing resource employment. The intricacies within TTSPs arise from their multimodal essence, characterized by numerous local and global optimal solutions. Simultaneously, the challenge is to strike a balance between comprehensive testing and the constraints of resources and time [1]. This multimodal complexity emerges from the various trade-offs among different objectives, rendering the quest for optimal solutions intricate and demanding. Furthermore, the design exploration spaces in TTSPs display heterogeneous multidimensionality and multimodality, introducing an additional layer of intricacy into the optimization endeavor [2].

Multimodal optimization algorithms work well for tackling the challenges of TTSPs. These algorithms can explore and make the best use of the different solutions available, finding various options that suit different needs. This helps decision-makers have more choices, considering things that regular design software might not catch. In TTSPs, the main objectives are usually to finish tests as quickly as possible (Makespan) and to use testing tools efficiently (mean workload). Making tests fast covers everything well, while lessening the workload uses resources better. These objectives can be in conflict, so it's important to find a balance to get the best solutions.

As the contribution of this study a new algorithm is developed to solve multimodal multiobjective TTSPs. Football Game Algorithm (FGA) first was proposed by the author of the current study in 2016 as a simple and efficient real valued optimization algorithm [3]. Later in 2022 in another study a multimodal version of FGA suitable was developed and applied to solve TTSPs [4]. In this study however, a further step is taken and a Multimodal Multiobjective (MM-FGA) FGA is developed to solve multiobjective TTSPs.

In the rest of the paper, we first briefly review the related work in this area then we discuss the structure of MM-FGA as well as the employed encoding scheme. The performance of the proposed algorithm is presented and discussed afterwards.

II. Related works

The TTSP is a type of puzzle that holds immense significance in complex systems like the automotive industry. In such systems, the final product's dependability hinges on these tests, while optimal scheduling influences production's speed, workload, and flexibility. In TTSP, it's not just about arranging tasks in order; there's also the consideration of how tasks are assigned to available tools. When certain tasks need multiple tools, these tools must work together. However, when a tool is used for one task, it can't be used for another at the same time. These unique TTSP characteristics set it apart from other scheduling problems like Flexible Job Shop Scheduling Problems (FJSPs) or Unrelated Parallel Machines Scheduling Problems (UPMSPs). The main goals in TTSPs are to finish tests quickly (Makespan) and use tools efficiently (Mean Workload) [5, 6].

Research on optimizing Test Task Scheduling Problems is expanding, but there's a lack of diversity in approaches, particularly in multimodal optimization. However, similar scheduling problems like Job Shop Scheduling Problems (JSSPs) and Parallel Machine Scheduling Problems (PMSPs) have been explored [6, 7]. Techniques from these studies can be adapted with slight adjustments for TTSPs [7]. Therefore, we not only delve into TTSP studies but also briefly review research on other related problems, particularly those aiming for multimodal optimization.

Perez et al. [8, 9], among the few, focused on identifying multiple solutions for Job Shop Scheduling Problems (JSSPs), which are typically multi-modal. They found that niching methods not only help locate multiple good solutions but also maintain diversity better than standard single-optimum genetic algorithms. In another recent study by Pan Zou et al. [10], a new algorithm was proposed for multimodal optimization of JSSPs. This algorithm combines k-means clustering with genetic algorithms (GA). The k-means algorithm clusters individuals based on machine sequence-related features, and then adapted genetic operators independently search for global optima within each cluster.

A recent study by Lu Hui et al. [11]introduced the Multi Center Variable Scale (MCVS) search algorithm for solving single and multi-objective problems, utilizing the multimodal property to design a new optimization strategy. MCVS emphasizes searching centers and neighborhoods. The study highlights the multimodality of scheduling problems [11]. Another study suggests that local search algorithms dependent on landscape smoothness are unsuitable for scheduling problems [12].

A Multi-strategy Fusion niching method has been proposed and used for solving several scheduling problems, including TTSP, FJSP, and PMSP [13]. Notably, this is among the few resources that presented results for solving TTSPs using the same multimodal single-objective optimization algorithm proposed in the paper.

In multiobjective scheduling optimization, notable studies include Zhang et al.'s hybrid PSO algorithm for solving the multi-objective FJSP [14]. This approach combines PSO's global exploration with a simulated annealing-based local search to enhance solution quality and convergence speed. The PSO explores the search space for global optima, and the SA algorithm refines these solutions, resulting in improved algorithm performance.

The hybrid TS algorithm discussed in [15] combines TS and a local search technique using the Pareto dominance-based neighborhood search (PDNS) algorithm. TS explores the search space and finds quality solutions, while PDNS refines these solutions to exploit the search space effectively.

Lu et al. introduced a method for solving the multi-objective automatic test task scheduling problem (ATTS) using a chaotic non-dominated sorting genetic algorithm (CNSGA) [7]. CNSGA, derived from NSGA, incorporates a chaotic map to enhance exploration and exploitation. This map generates random numbers for genetic operations, promoting diversity and randomness in selection, crossover, and mutation. This approach aims to prevent early convergence and improve the search process.

The Multi-Objective Test Task Scheduling Problem (MTTSP) is a complex challenge in software testing, involving the scheduling of test tasks on resources to optimize objectives like minimizing test time and maximizing resource use. Lu et al. [16]propose a variable neighborhood multi-objective evolutionary algorithm based on decomposition (MOEA/D) to solve this problem. This algorithm adapts by dynamically selecting different neighborhood structures for subproblems, catering to the evolving problem landscape and exploring diverse search areas.

However, in a subsequent work [17], Lu et al. argue that for discrete problems like TTSPs, Pareto-based methods are more fitting. They introduce a multi-objective evolutionary algorithm based on Pareto prediction (PP-MOEA) for solving the Automatic Test Scheduling Problem (ATSP). This method involves two stages: in the first, a prediction model is trained using high-quality solutions from a conventional MOEA, guiding the search. In the second stage, a guided search operator is devised to create solutions based on the predicted Pareto front, and a modified NSGA-II algorithm selects solutions. Results demonstrate the proposed algorithm's superior convergence and diversity, outperforming MOEA/D and NSGA-II, by generating a more diverse and well-distributed set of solutions along the Pareto front.

III. Problem Definition

TTSPs fall within the realm of combinatorial optimization problems, a subset of discrete optimization problems. The decision variables involve permutations or combinations selected from a finite set of discrete options. The mathematical representation of TTSP is drawn from [6, 17] , and readers seeking further details are directed to these sources for brevity in focusing on the paper's main contribution.

IV. Multimodal Single-objective Football Game Algorithm

The baseline in the development of MM-FGA would be the proposed Multimodal FGA (M-FGA) in [4]. So, we briefly review the main components of the M-FGA here and based on that develop and discuss the enhancements for MM-FGA.

The main algorithmic steps in the M-FGA are as follows:

1. Initialization using uniform rejection sampling
2. Elitism using Phenotypic Distributed Elitism (PDE)
3. Positioning (analogous to random walk/evolution)
    a. Substitution
    b. General positioning
4. Evaluation and ranking
5. Check for termination criteria
    a. Not met: go to step 2
    b. Met: continue to step 6

6. Postprocessing and subset selection from the Coach Memory (CM)

We employ Uniform Rejection Sampling as the initialization step for the algorithm. Unlike a basic uniform sampling, this approach rejects new individuals that are positioned very closely to already sampled points. An initial rejection radius is defined, which can be computed by dividing the available search volume by N_p hyper-spheres around each sampled point. The volume of a D-dimensional hyper-sphere can be calculated using the formula [18]:

$$V_{hs} = \frac{2\pi^{\frac{D}{2}}}{D\Gamma(D/2)} r^D \tag{1}$$

Therefore, the initial rejection radius r_0 based on available search volume can be found using (2) and (3) respectively.

$$r_0 = \left(\frac{D}{2}\frac{\Gamma(D/2)}{\pi^{\frac{D}{2}}}\frac{V_a}{N_p}\right)^{\frac{1}{D}} \tag{2}$$

$$V_a = \prod_{i=1}^{D}(UB_i - LB_i) \tag{3}$$

Where UB and LB representing upper and lower bound of each variable respectively. In the M-FGA the step-size will be reduced based on an exponential decay by θ decay rate and r_0 as the initial step-size.

$$\alpha(t) = \alpha_0(\theta)^t \tag{4}$$

In the M-FGA a distant CM strategy has been used to increase the diversity and the stability of the found solutions up to the end of the run. For this purpose, a decreasing pairwise Mahalanobis distance metric between the CM positions is considered. It means that there would be a decreasing limitation for the distance between every 2 positions in the CM list. Using evolutionary algorithm analogy and in comparison to the greedy elitism in classic FGA we call this mechanism the Phenotypic Distributed Elitism (PDE). It helps the algorithm to have a better coverage over the search space in the exploration phase and consequently demonstrates an effective implicit basin identification and improves the algorithm's performance. This distance limit resembles niche radius in other methods however its functionality is different here. In M-FGA the niche radius refers to the distance that can be covered by random walks from the previous position considering a certain confidence level. So, in order to minimize the search overlaps between two adjacent niches, pairwise distance limit between to CM samples is considered to be double the niche radius. This approach enables the population to allocate a greater search effort towards exploring challenging basins characterized by smaller or complex shapes, which are comparatively more difficult to uncover than larger, smoother basins. In PDE, the maximum distance between elite samples is determined as a multiple of the step-size and dynamically decreases as the optimization iteration progresses. This distance parameter should be fine-tuned to ensure optimal coverage of random walks around each CM sample. More detailed information can be found in [4].

M-FGA differs from classic FGA by removing the ball owner effect and attacking strategy, which enhance solution instability and local search intensity. In Modified FGA, player movements are simplified to Brownian motion around their prior positions for positioning or around a chosen CM position using Fitness Proportionate Selections for substitutes.

General positioning:

$$x_i(t) = x_i(t-1) + \alpha(t)\mathcal{N}(0, I_D);\ \ \forall\, i = 1,2,\dots,N_p - N_s \tag{5}$$

Players' substitution:

$$x_i(t) = CM^j_{position} + \alpha(t)\mathcal{N}(0, I_D);$$

$$j \in_R [1, N_t]\,, \forall\, i = (N_p - N_s) + 1, \dots, N_p \tag{6}$$

Where N_p is the number of the population, D is the problem's dimension, N_t is the number of tactics or CM-size while N_s is the number of substitutions in each iteration, and $\mathcal{N}(0, I_D)$ is a D-dimensional random vector drawn from the standard normal distribution.

V. Multimodal Multi-objective Football Game Algorithm

Building upon the foundational structure of M-FGA, additional elements are introduced into the algorithm to effectively handle the multiobjectivity challenges within multiobjective TTSPs. The initialization strategy remains unchanged, employing the same rejection sampling method. However, in contrast to M-FGA, solutions are ranked using the non-dominated sorting method. The presence of multiple solutions in each rank aligns well with FGA's elitism strategy of storing diverse solutions in the CM. Solutions of the same non-dominated rank hold equal priority, eliminating the need for measures like crowding distance in TTSPs. These solutions are sorted in the CM based on their frequencies in the objective space. To ensure a balanced distribution of frequencies among solutions, lower frequency CM samples are prioritized as base vectors for substitution. It worth noting that solution frequency in the objective space refers to the count of distinct solutions in the decision space with the same objective values.

Elite solutions are stored in the CM using PDE, with distinctions from M-FGA. In MM-FGA, the Hamming distance, due to the discrete search space's nature, replaces the Mahalanobis distance metric in PDE. The size of the CM (N_t) is adaptive, ranging from a user-set minimum to the maximum number of rank-one non-dominated solutions.

It is discussed in [4] that how the IES and Normalized Factor Random Key (NF-RK) encoding scheme will partition the search space to a discrete set of decision tiles each representing a unique combination of tasks and machines. Considering the clear distinction between continuous and discrete landscapes, the normal distribution used for random walks is replaced by a uniform distribution. This change aligns better with the decision tile shapes. Similarly, for base vectors in substitution, a frequency proportionate selection method is employed. Solutions with lower frequencies are given higher chances for selection as base vectors. This transformation also extends to general positioning, where uniform distribution replaces the previous normal distribution. These adjustments enhance the compatibility of the algorithm with discrete search spaces.

To prevent the step-size from becoming too small, which could hinder effective mutation power for escaping converged decision tiles, we introduce a minimum step-size to the algorithm. This minimal value is derived from the decision tile's size and shape. It's important to note that this minimum step-size doesn't serve as an algorithm termination criterion. The search process is divided into two phases: the Exploration phase initiates with initialization and continues until the step-size reaches its minimum, followed by the Exploitation phase, encompassing the remaining search iterations. Throughout the Exploitation phase, the step-size maintains its minimum value. The user determines the balance between Exploration and Exploitation by specifying when the search should transition to the Exploitation phase, thus influencing the step-size reduction rate calculation. Our observation indicates that when the algorithm reaches the minimum step-size, it proceeds to exploit the local region, generating new local solutions. This phase significantly enhances the final search outcome. This behavior can be attributed to the randomness introduced by direction vectors drawn from a uniform distribution and the highly noisy landscape of the TTSPs.

Table 1 is to elucidate the distinctions and resemblances between the proposed MM-FGA and M-FGA.

TABLE I. CONTRIBUTION OF THE MM-FGA IN COMPARISON TO M-FGA

Algorithms	M-FGA	MM-FGA
Initialization	Uniform Rejection Sampling	Uniform Rejection Sampling
Sorting	Ascending order of the population's fitness values (minimization problem)	Non-dominated sorting
Elitism	- PDE with Mahalanobis distance limit - Maximum CM-size of N_t	- PDE with Hamming distance limit - Adaptive CM-size with minimum of N_t
Substitution	- Normal distribution drawn random walk - Fitness proportionate selection of the base vector - Step-size reduction up to the end of run	- Uniform distribution random walk - Frequency proportionate selection of the base vector - Minimum step-size strategy for a distinct exploitation phase
General positioning	- Normal distribution drawn random walk	- Uniform distribution random walk - Minimum step-size strategy for a distinct exploitation phase

VI. MULTIMODAL MULTI-OBJECTIVE OPTIMIZATION OF TTSP

In this part of the study the final implementation of MM-FGA on the large scale real-world TTSPs is presented. These specific TTSPs involve varying scales: 20t-8m, 30t-12m, 40t-12m, and 50t-15m. Following the precedent set by other investigations dealing with the same problem set [7, 17], we adopt the search budgets detailed in Table 2 for each case.

TABLE II. ALLOCATED SEARCH BUDGET TO EACH PROBLEM

TTSP	Population	Generation
20t-8m	70	120
30t-12m	100	250
40t-12m	100	250
50t-15m	100	250

For the purpose of comparison study Four powerful algorithms for Multimodal Multiobjective Optimization Problems (MMOPs) are selected: Tri-MOE-TA&R [19], MP-MMEA [20], MO-Ring-PSO-SCD [21], DN-NSGA-II [22]. Additionally, we included NSGA-II [23] as a legacy algorithm among the EMOAs. We use the standard version of the algorithms in PlatEMO and keep the parameter settings as default [24].

The algorithm's performance is assessed using the HV indicator to identify the dominant Pareto set and gauge convergence in the objective space. Additionally, the count of distinct solutions (N) in the decision space is employed as an effective metric to measure and compare the algorithms' capability in addressing the problems' multimodality.

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The search budget is balanced between Exploration and Exploitation, allocating 70 percent for Exploration and 30 percent for Exploitation. The step-size reduction aims to reach the minimum step-size within 84 generations for the 20t-8m TTSP and within 175 generations for the other problems. Consequently, reduction rates of $\theta = 0.8$ for 20t-8m TTSP and $\theta = 0.91$ for the remaining cases are calculated. $N_t = 0.9$ is uniformly chosen across all scenarios to maximize the population's spread over the search landscape.

we solve each problem for 51 times using each of the algorithms. The results of solving these practical large scale TTSPs by MM-FGA and five other algorithms are reported in Table 3 to Table 6.

The results for the 20t-8m TTSP indicate that MM-FGA demonstrates acceptable performance based on the HV measure compared to the other algorithms. While it may not have the highest mean HV, it shows significant overlap in performance with NSGA-II and MP-MMEA. It's important to note that HV measures the spread of non-dominated solutions in the objective space, reflecting the algorithm's capability in finding quality multiobjective solutions. On the other hand, N assesses an algorithm's ability to discover diverse solutions in the decision space. The obtained N values clearly demonstrate MM-FGA's superiority over the other five algorithms. The mean N value of 45.47 in Table 3 exceeds the third quartile of any other algorithm, showcasing MM-FGA's excellence in uncovering varied solutions in the decision space, while maintaining HV values within a comparable range with the other algorithms.

The statistical outcomes presented in Table 4 for the 30t-12m TTSP demonstrate comparable performance between MM-FGA and the other algorithms. In this case, there's a slight advantage in the mean HV for MM-FGA, falling within the same range as NSGA-II and MP-MMEA. MM-FGA stands out with a mean N value of 48.94, outperforming the other algorithms in this metric. DN-NSGA-II also remains competitive, securing the second position with a mean N of 43.82 and a noticeable overlap.

TABLE III. PERFORMANCE OF MM-FGA ON 20T-8M TTSP IN COMPARISON TO THE OTHER ALGORITHMS

	HV				**N**			
Algorithm	Mean	Var.	Min	Max	Mean	Var.	Min	Max
MM-FGA	2.4955e4	3.6641e4	2.4528e4	2.5410e4	**45.4706**	320.4541	**22**	**97**
NSGA-II	2.5024e4	3.2056e4	2.4691e4	**2.5448e4**	27.0980	31.5702	16	40
DN-NSGA-II	2.4854e4	4.5736e4	2.4415e4	2.5305e4	29.6078	52.6431	17	45
Tri-MOE-TA&R	2.4795e4	5.9239e4	2.4269e4	2.5379e4	29.3333	96.1067	13	51
MP-MMEA	2.5026e4	2.4822e4	**2.4692e4**	2.5406e4	20.1765	**11.8282**	12	29
MO-Ring-PSO-SCD	**2.5093e4**	**2.2783e4**	2.4434e4	2.5093e4	24.8627	24.7608	14	38

TABLE IV. PERFORMANCE OF MM-FGA ON 30T-12M TTSP IN COMPARISON TO THE OTHER ALGORITHMS

	HV				**N**			
Algorithm	Mean	Var.	Min	Max	Mean	Var.	Min	Max
MM-FGA	**6.7504e4**	2.2197e5	6.6549e4	**6.8929e4**	**48.9412**	224.6965	**28**	**119**
NSGA-II	**6.7539e4**	2.3809e5	6.6458e4	6.8580e4	39.8431	56.6949	24	56
DN-NSGA-II	6.7327e4	2.5672e5	6.6243e4	6.8461e4	43.8235	98.7882	**28**	69
Tri-MOE-TA&R	6.7162e4	2.5093e5	6.6058e4	6.8291e4	38.6863	153.2196	21	85
MP-MMEA	6.7423e4	2.1866e5	**6.6608e4**	6.8841e4	32.9216	**37.0737**	26	50
MO-Ring-PSO-SCD	6.6895e4	**1.1105e5**	6.6124e4	6.7726e4	40.0000	56.5200	27	57

TABLE V. PERFORMANCE OF MM-FGA ON 40T-12M TTSP IN COMPARISON TO THE OTHER ALGORITHMS

	HV				**N**			
Algorithm	Mean	Var.	Min	Max	Mean	Var.	Min	Max
MM-FGA	**1.2017e5**	3.2266e5	**1.1887e5**	1.2131e5	**54.0874**	224.6337	30	**102**
NSGA-II	**1.1996e5**	3.5898e5	**1.1872e5**	1.2141e5	51.0588	103.6965	25	74
DN-NSGA-II	1.1921e5	3.0285e6	1.1134e5	1.2114e5	48.4706	164.6941	8	70
Tri-MOE-TA&R	1.1980e5	4.2707e5	1.1865e5	**1.2167e5**	39.0196	103.8596	20	59
MP-MMEA	1.1980e5	6.7636e5	1.1814e5	**1.2152e5**	42.5490	35.3725	28	51
MO-Ring-PSO-SCD	1.1875e5	**2.3520e5**	1.1787e5	1.1976e5	42.0588	**21.9365**	**31**	51

TABLE VI. PERFORMANCE OF MM-FGA ON 40T-12M TTSP IN COMPARISON TO THE OTHER ALGORITHMS

	HV				**N**			
Algorithm	Mean	Var.	Min	Max	Mean	Var.	Min	Max
MM-FGA	1.8185e5	1.3044e6	1.7866e5	1.8436e5	**66.2745**	580.8431	**36**	**135**
NSGA-II	**1.8312e5**	1.6417e6	1.8009e5	1.8571e5	57.1176	111.9859	35	78
DN-NSGA-II	1.8216e5	2.0885e6	1.7794e5	1.8461e5	55.0980	158.9702	32	82
Tri-MOE-TA&R	1.8261e5	1.6560e6	1.7831e5	1.8573e5	42.9020	118.3702	25	81
MP-MMEA	1.8234e5	1.1774e6	**1.8022e5**	**1.8604e5**	37.6863	32.0596	21	48
MO-Ring-PSO-SCD	1.7779e5	**7.4616e5**	1.7606e5	1.7987e5	23.2941	**11.1318**	16	32

Regarding the 40t-12m TTSP, MM-FGA achieves a superior mean HV compared to NSGA-II in second place as presented in Table 5. However, when observing the HV distribution across the algorithms, their performances appear relatively consistent, with the exception of MO-PSO-CD, which exhibits the lowest mean HV. The distribution of N for MM-FGA, NSGA-II, and DN-NSGA-II shows a close resemblance, with MM-FGA slightly edging ahead in terms of mean N due to a few outliers around 100.

The final problem involves 50 tasks and 15 machines, making it a highly multimodal challenge with 50 decision variables. Table 6 presents that NSGA-II achieves the highest mean HV value. However, even though MM-FGA doesn't secure the best mean HV, it still performs within the range of top multiobjective algorithms. In terms of the number of solutions in the decision space, MM-FGA outperforms all other algorithms in mean, minimum, and maximum N. This highlights MM-FGA's ability to find and maintain diverse solutions in the decision space, even in such a complex scenario.

VII. CONCLUSION AND FUTURE WORKS

We employed MM-FGA to address the four real-world TTSPs, concentrating on optimizing makespan and mean workload. To accommodate the NF-RK encoding scheme, we integrated a modified minimum step-size approach into the MM-FGA algorithm. The performance of MM-FGA was then compared against five alternative algorithms, namely NSGA-II, DN-NSGA-II, Tri-MOE-TA&R, MP-MMEA, and MO-Ring-PSO-CD. In general, MM-FGA exhibited performance similar to the most effective multiobjective algorithms in handling the multiobjectivity challenge in optimization problems. Notably, MM-FGA outperformed the other five algorithms in terms of the number of solutions discovered in the decision space. This underscores MM-FGA's exceptional ability to tackle the complexity of multimodality in optimization problems. MM-FGA has proven to be a robust algorithm for addressing multimodal multiobjective discrete problems, particularly TTSPs, and it also holds potential for further enhancements in future research.

In the realm of potential future research, the control of step-size management emerges as a critical determinant in achieving an equilibrium between exploration and exploitation, thus shaping the algorithm's overarching efficacy. However, an avenue ripe for exploration involves delving deeper into a more intricate approach to step-size adaptation during the course of the search. This could encompass the exploration of meta-optimization techniques applied to this parameter, enabling the implementation of a more refined and sophisticated step-size adjustment strategy. Such an endeavor holds the promise of potentially elevating the algorithm's performance to new heights through a more nuanced and optimized mechanism.

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