

# Task assignment and path planning of multiple unmanned aerial vehicles using Integer Linear Programming

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**Abstract**— In this paper, we propose a strategy to enhance the performance of task assignment and path planning in applications of distributed multiple unmanned aerial vehicles (multi-UAV). Multi-UAVs are made up of many small UAVs with limited mission resources. They can operate autonomously, appropriately, and universally. UAV swarm task coordination and resource allocation can be achieved with reasonable allocation of UAV tasks and resources according to the UAV region and its own performance.

Based on the in-depth research of the traditional auction algorithms, this work proposes a method that can improve multi-UAV task allocation efficiency, namely the Integer Linear Programming (ILP). We are proposing ILP formulation for numerous drones and tasks in an environment that enables the completion of tasks. We also measure the computational time requirements of the proposed ILP approach with various number of drones and tasks, so that the limit of this approach can be identified for practical application. This research overcomes the difficulties between task allocation and path planning. Compared with auction-based algorithms this technique can better complete allocation results and reduce resource consumption. It's expected that the simulation results will show that the algorithm is effective in computing and task execution efficiency.

**Keywords**— Task assignment, path planning; Multi-UAV; Auction algorithm; ILP Algorithm.

## I. INTRODUCTION

Task assignment and path planning are critical aspects of coordinating multiple Unmanned Aerial Vehicles (UAVs) in various applications, ranging from surveillance and reconnaissance to disaster response and package delivery. The efficient allocation of tasks and optimal path planning for UAVs can significantly enhance their performance, reduce mission completion time, and improve overall mission success rates. In recent years, researchers have focused on developing intelligent algorithms and optimization techniques to address these challenges. One such approach is the utilization of Integer Linear Programming (ILP) for task assignment and path planning of multiple UAVs.

The ILP framework provides a mathematical modeling technique to formulate complex optimization problems

involving discrete decision variables, objective functions, and a set of constraints. By representing the task assignment and path planning problem as an ILP model, it becomes possible to find an optimal or near-optimal solution, considering various factors such as task priorities, resource constraints, and environmental conditions. The ILP approach offers the advantage of rigorous mathematical optimization, allowing for systematic exploration of the solution space.

The task assignment problem involves determining which tasks should be assigned to each UAV, considering factors such as task importance, UAV capabilities, and mission requirements. Path planning, on the other hand, focuses on finding optimal routes or trajectories for the UAVs to navigate from their current locations to the assigned tasks. The path planning problem is challenging due to several factors, including limited UAV resources, complex and uncertain environments, and potential conflicts with other UAVs or obstacles. ILP-based path planning models can incorporate these constraints and generate collision-free paths that optimize criteria such as travel distance, mission completion time, or energy consumption.

The integration of task assignment and path planning within an ILP framework enables a comprehensive optimization approach for multiple UAVs. By jointly considering task assignment and path planning, the ILP model can account for dependencies and interactions between tasks and paths, leading to more efficient and coordinated UAV operations.

Several research studies have investigated the application of ILP in the context of task assignment and path planning for multiple UAVs. Li et al. (2017) [1] proposed an ILP-based framework for cooperative task allocation among UAVs, considering task types, deadlines, and UAV capabilities. Wang et al. (2019) [2] presented an ILP formulation for joint task assignment and path planning, aiming to minimize the total mission completion time. Yang et al. (2020) [3] developed an ILP-based approach for multi-UAV surveillance task assignment, incorporating multiple objectives and constraints.

Moreover, the use of ILP in UAV task assignment and path planning has been extended to specific domains and applications. For instance, Chen et al. (2018) [4] focused on

the allocation of sensing tasks to UAVs in a target tracking scenario, considering both sensing and communication constraints. Guo et al. (2020) [5] proposed an ILP-based approach for task assignment and path planning of UAVs in precision agriculture applications, optimizing crop coverage and minimizing travel distance. These studies demonstrate the versatility and effectiveness of ILP in addressing task assignment and path planning challenges in various UAV applications.

In , task assignment and path planning of multiple UAVs play a crucial role in optimizing mission performance and resource utilization. The utilization of ILP provides a powerful framework for modeling and solving these complex optimization problems. By integrating task assignment and path planning within an ILP formulation, it becomes possible to achieve coordinated and efficient UAV operations in environments. The following sections of this paper will delve into the details of ILP-based approaches, algorithms, and case studies related to task assignment and path planning for multiple UAVs.

## II. LITERATURE REVIEW

Drones and unmanned aerial vehicles (UAVs) were first used in battle against Austrian troops using balloon carriers in 1849, marking the beginning of the usage of drones and UAVs. Drones were first utilized largely for military operations. But throughout time, a lot of study has been done, resulting in the use of this technology for a number of different activities. Drones are now often used for package delivery, cargo and passenger transportation, and even in space missions to autonomously carry supplies to space stations.

For real-time mission planning and dynamic agent-to-task assignment for UAV swarms, a scalable and adaptable architecture is proposed [6]. In this design, a local coordinator known as Agent Mission Planner (AMP) oversees carrying out these activities with an asynchronous communication with the GMP. Global Mission Planner (GMP) is responsible for assigning and monitoring various high-level missions. The Polygon Visiting Many Traveling Salesman Problem, also known as the task assignment for cooperative UAVs, is solved using two alternative evolutionary fuzzy clustering techniques that are more effective than k-means and c-means clustering. To cluster the search space, one method compares the distance traveled by each UAV, and the other [7] employs a cost function that approximates the traveled distance.

Although several collision avoidance approaches have been reported, there is a lack of highlighting the key components shared by these approaches. In this subject of [8] to provide researchers with a state-of-the-art overview of various approaches for multi-UAV collision avoidance. [9] Aim to maximize the system capacity by jointly optimizing the sub channel assignment, the uplink transmit power of IoT nodes, and the flying heights of UAVs. A quadrotor unmanned aerial vehicle (UAV) should have the ability to perform real-time target tracking and path planning simultaneously even when the target enters unstructured scenes, such as groves or forests. [10] To accomplish this task, a novel system framework is designed and proposed to accomplish simultaneous moving target tracking and path planning by a quadrotor UAV with an onboard embedded

computer, vision sensors, and a two-dimensional laser scanner [11].

The area between the UAV group range and the group communication range is called the insecurity range and, in the region, multi-UAV communication can cause serious information leakage. To resolve this problem [12] consider two aspects, namely, cooperative control and secure communication. Unmanned aerial vehicle (UAV) path planning problem is an important component of UAV mission planning system, which needs to obtain optimal route in the complicated field. To solve this problem, a novel hybrid algorithm called HSGWO-MSOS is proposed by combining simplified grey wolf optimizer (SGWO) and modified symbiotic organisms search (MSOS) [13]. In this work the Integer Linear Programming is introduced. ILP is a form of optimization problem when the objective function and equations are linear, the variables are integer values, and the optimization problem [14].

## III. MATHEMATICAL MODELLING

The centralized auction algorithm is a suitable approach for solving the task assignment problem when there are few agents, and the network topology is well-connected. It involves a central station that distributes global information about available resources and assignment results to all participating bidders. However, as the number of agents increases or when agent systems operate on unreliable networks, the communication cost of maintaining a central station can become prohibitive. Additionally, the scalability of the centralized algorithm may be affected by the network topology. To address these challenges and improve task allocation efficiency for multi-UAV systems, this work proposes the use of an iterative method known as the integer linear programming (ILP) algorithm. The ILP algorithm enhances the allocation process by considering the integer constraints inherent in the task assignment problem. By formulating the problem as an ILP, the algorithm can optimize the allocation of tasks among the UAVs, leading to improved efficiency in task allocation.

Furthermore, to enhance the daily management of computing and communication resources in UAVs, this paper overcomes the issues arising from the data coupling between task allocation and path planning. It introduces a decentralized task allocation algorithm, where each UAV re-evaluates and verifies the assigned tasks within the task allocation cycle. This approach enables the UAVs to identify and rectify any unreasonable task allocation results, ensuring better coordination and overall performance in task execution.

To overcome the problem, we have proposed the algorithm. When comparing the Integer Linear Programming (ILP) algorithm with other algorithms for multiple task assignment in UAVs, the key considerations are efficiency and scalability. ILP algorithms excel at handling complex optimization problems efficiently, providing optimal or near-optimal solutions. They are capable of handling large-scale task assignment problems with a high number of tasks and drones. In contrast, other algorithms may struggle to achieve the same level of optimization and scalability. ILP algorithms offer a mathematical framework for formulating the task assignment problem, considering various constraints, and finding optimal solutions, making them a powerful choice for multiple task assignment in UAV systems.

We consider a UAV network which consists of  $K$  number of drones and  $N$  number of tasks. The distance between drone and task  $i$  and  $k$  is denoted as  $\text{dist}DT_{i,k}, \forall i \in \{1, \dots, N\}$  and,  $\forall k \in \{1, \dots, K\}$ . The distance between any two task  $i$  and  $j$  is denoted as  $\text{dist}TT_{i,j}, \forall i \in \{1, \dots, N\}$  and,  $\forall j \in \{1, \dots, N\}$ . In this regard, we formulate the ILP for the multiple task assignment in multi-UAV network as follows. The decision variables considered for the ILP formulation are as below:

$$x_{i,j} \begin{cases} 1, \text{ if drone travels from task } i \text{ to task } j, \forall i, j \in \{1, \dots, N\} \text{ and } i \neq j \\ 0, \text{ otherwise} \end{cases}$$

$$y1_{k,j} \begin{cases} 1, \text{ if drone travels from its origin } k \text{ to task } j, \forall k \in \{1, \dots, K\} \text{ and } \\ \forall j \in \{1, \dots, N\} \\ 0, \text{ otherwise} \end{cases}$$

$$y2_{i,k} \begin{cases} 1, \text{ if drone travels from task } i \text{ to origin } k, \forall i \in \{1, \dots, N\} \text{ and } \\ \forall k \in \{1, \dots, K\} \\ 0, \text{ otherwise} \end{cases}$$

$u_i$ : time instant at which task  $i$  is visited,  $\forall i \in \{1, \dots, N\}$ .

The objective function is given as.

$$\text{minimize } \sum_{j=1}^N \sum_{k=1}^K y1_{k,j} \text{dist}DT_{j,k} + \sum_{j=1}^N \sum_{i=1, i \neq j}^N x_{i,j} \text{dist}TT_{i,j} + \sum_{i=1}^N \sum_{k=1}^K y2_{i,k} \text{dist}DT_{i,k}.$$

The above objective is to minimize the total distance that a drone(s) takes to complete the task starting from its origin and returning to its origin. The term  $\sum_{j=1}^N \sum_{k=1}^K y1_{k,j} \text{dist}DT_{j,k}$  shows that drone is starting from its origin  $k$  and moving to task  $j$ . The term  $\sum_{j=1}^N \sum_{i=1, i \neq j}^N x_{i,j} \text{dist}TT_{i,j}$  represents that the drone travels from task  $i$  to task  $j$ . The term  $\sum_{i=1}^N \sum_{k=1}^K y2_{i,k} \text{dist}DT_{i,k}$  represents the return of drone(s) after completing the task.

Following are the constraints that are considered for the above ILP objective.

$$\sum_{j=1}^N \sum_{k=1}^K y1_{k,j} \leq K \quad (1)$$

In the task assignment scenario, the constraint dictates that the number of drones commencing from their origin to execute a task must not exceed the limit of  $K$  drones. This limitation ensures efficient utilization of resources and maintains a manageable workload distribution. By adhering to this constraint, the system can optimize the allocation of tasks among the available drones while considering the limitations of their origin locations.

$$\sum_{k=1}^K y1_{k,j} + \sum_{i=1, i \neq j}^N x_{i,j}, \forall j \in \{1, \dots, N\} \quad (2)$$

In the task allocation process, it is required that each task is assigned to drone(s) in such a way that they arrive at the task location exactly once. This condition ensures that the tasks are efficiently distributed among the drones, avoiding duplication or omission. By enforcing this constraint, the system can maintain a reliable and accurate execution of tasks, optimizing the overall performance and preventing any inconsistencies or inefficiencies in the assignment process.

$$\sum_{k=1}^K y2_{i,k} + \sum_{i=1, i \neq j}^N x_{i,j}, \forall i \in \{1, \dots, N\} \quad (3)$$

In addition to ensuring that each task is assigned to drone(s) and arrives exactly once, it is also crucial that the

drone(s) leave the task location exactly once. This requirement guarantees that the drones complete their assigned tasks and move on to subsequent assignments without any redundant or missed operations. By enforcing this constraint, the system maintains a streamlined workflow, minimizing delays and maximizing the efficiency of task execution. It also enables proper resource allocation and scheduling, allowing for effective coordination among the drones in the system.

$$\sum_{j=1}^N y1_{k,j} = \sum_{i=1}^N y2_{i,k}, \forall k \in \{1, \dots, K\} \quad (4)$$

As part of the task assignment process, it is essential that each drone, after completing its assigned task, returns to its original starting point. This requirement ensures that the drones maintain a closed loop trajectory, completing their mission and returning to their designated origin. By enforcing this constraint, the system can optimize the utilization of resources and ensure the efficient operation of the drones. It also allows for better planning and coordination of subsequent tasks, as the drones are available at their designated starting locations for future assignments.

$$u_j - u_i > = 1 - N(1 - x_{i,j}), \forall i, j \in \{1, \dots, N\} \text{ and } i \neq j \quad (5)$$

The sub-tour elimination constraint, derived from the Miller-Tucker-Zemlin method, is a crucial component in optimizing the task assignment process. This constraint ensures that the solution does not contain any sub-tours, meaning that all drones' routes form a single connected tour. By imposing this constraint, the system prevents inefficient and overlapping routes, leading to a more optimal and streamlined allocation of tasks among the drones. This constraint enhances the overall efficiency and effectiveness of the task assignment algorithm, reducing redundancy and improving the overall performance of the system.

The combination of the five constraints ensures the comprehensive and efficient completion of tasks by the drones. By enforcing these constraints, we guarantee that each drone visits every task, leaving no task unattended. Moreover, the constraints optimize the assignment process by considering the shortest path condition, resulting in streamlined routes and minimizing overall travel distances. This approach not only enhances the efficiency of task allocation but also maximizes resource utilization, ultimately leading to improved performance and successful task execution in the system.

#### IV. SIMULATION AND RESULTS

##### A. One drone with multiple task

The first result shows when the number of drones is one and tasks are six how the drone will work and complete its journey by keeping the shortest path condition in its mind.

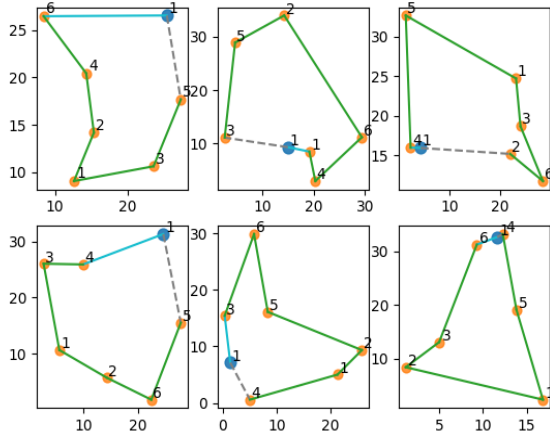


Figure 1. One drone and six tasks.

In Figure 1, the orange dots in Figure 1 indicate the jobs that are yet unfinished, whereas the blue dot in Figure 1 depicts the drone stationed at its origin. The drone carefully follows the green line as it navigates around the environment to complete each duty set to it. The drone will successfully finish its journey thanks to its meticulously planned route, which will take the shortest route possible to save time and resources. The dotted line depicts the drone's return path to its starting point once it completes a mission successfully. This return trip marks the end of the drone's cycle for assigning tasks, enabling it to reboot and be ready for the subsequent batch. The dotted line draws attention to the smooth transition from mission completion to repositioning, emphasizing the drone's quick and efficient return to its starting location. Figure 1's depiction of the coordinated movement of drones shows how they efficiently visit and carry out duties while maintaining an optimum workflow. Drones are useful assets in a variety of operational settings because the strategic planning and execution they display assure optimal productivity and resource utilization.

*B. Multiple drones and multiple tasks*

The presented results illustrate the coordinated efforts of multiple drones and multiple tasks, showcasing how the drones efficiently complete their journeys by following the shortest path. Each drone, represented by a blue dot at its origin, dynamically analyzes the task allocation and strategically plans its route. The drones navigate through the terrain, indicated by the green lines, carefully visiting their assigned tasks represented by the orange dots. By continuously optimizing their paths, the drones minimize travel distances, ensuring efficient task completion. Once a drone finishes a task, it seamlessly transitions to the next, consistently considering the shortest path to maintain productivity. The synchronized movements of the drones result in the timely completion of all tasks. The dotted lines portray the drones' return paths, leading them back to their respective origins after accomplishing their missions. These results exemplify the effectiveness of utilizing multiple drones, each following the shortest path, to maximize productivity and streamline operations in complex task

scenarios.

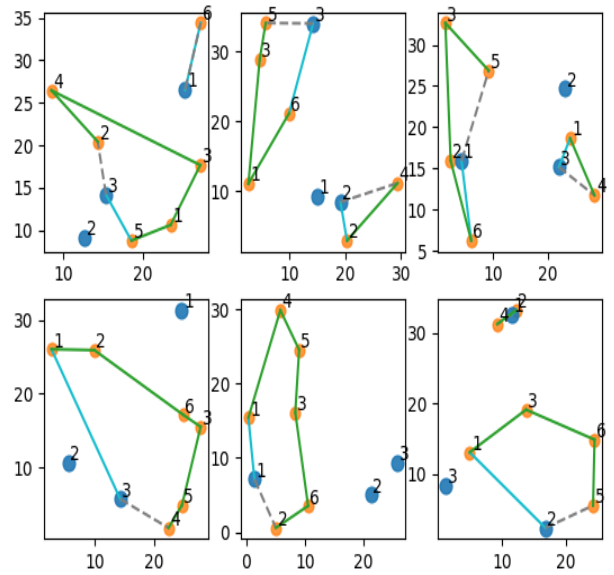


Figure 2. Three drones and six tasks.

Figure 2 offers a thorough illustration of the drones' effective work distribution method. The closest drone to the job is given the assignment, ensuring the quickest route is taken. The drones quickly and accurately carry out their assigned jobs as they move through the designated zones. The drones immediately return to their separate starting points when a task is satisfactorily finished, prepared to start their next assignment. Maximum productivity is made possible by this methodical approach, which reduces overall trip time and optimizes the use of resources. The coordinated workflow shown in Figure 2 demonstrates how drones may be easily incorporated into a variety of operating contexts, resulting in increased productivity and simplified procedures.

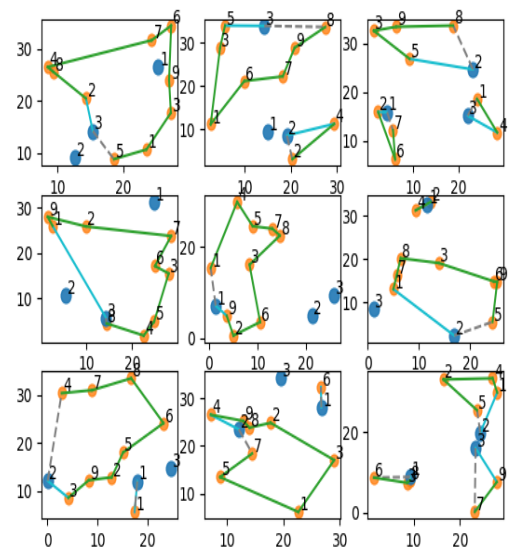


Figure 3. Three drones and nine tasks.

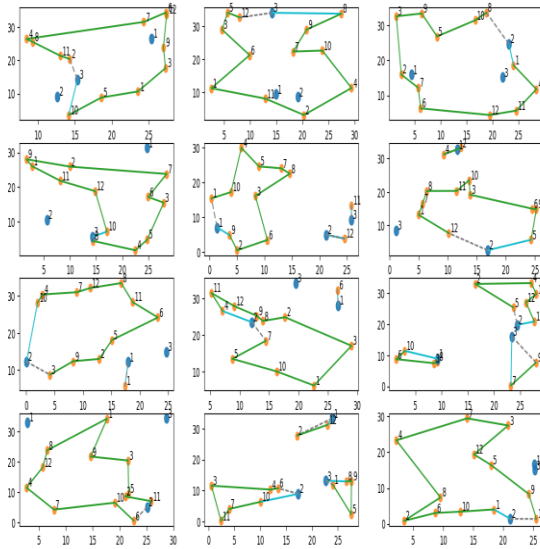


Figure 4. Three drones and twelve tasks.

#### CONCLUSION

The proposed study aims to compare the performance of the existing algorithm with the ILP approach. Because the ILP approach is expected to provide a globally optimal solution, which can guarantee the optimality of the solution.

Overall, the study is expected to demonstrate the ILP over the existing algorithms in terms of efficiency and overall cost. By comparing the results of these four algorithms, the study will contribute to the development of optimum path planning and task assignment for multi-UAVs.

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