An Efficient Channel Estimation Technique for Hybrid IRS Assisted Multiuser Wireless Communication System Based on Tensor Modelling

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***Abstract*—We address the challenging problem of channel estimation in a narrowband multiuser communication that incorporates intelligent reflecting surfaces (IRS). To overcome the limitations of the two-hop channel in passive IRS system, we adopt a Hybrid IRS architecture, which involves integrating active sensors into a few elements of the IRS. By exploiting the sparse characteristics of the channel, we utilize parallel factor analysis (PARAFAC) tensor models to represent the training signals. We propose an algebraic algorithm to estimate the channel and evaluate the normalized mean square error (NMSE) between the estimated and actual parameters. Additionally, we explore the uniqueness of the canonical polyadic decomposition (CPD) and analyze the essential conditions necessary for achieving accurate channel estimation. Through simulation studies, we validate the effectiveness of our proposed algorithm in estimating channels in a multiuser IRS system.**

***Index Terms*—Channel estimation, PARAFAC tensor completion, BALS algorithm, Hybrid IRS**

I. INTRODUCTION

The future of wireless communication looks interesting with the new use cases and requirements which are quite challenging for the future sixth generation (6G) systems. Throughout the evolution of modern wireless systems, the propagation medium has remained inherently stochastic, adding complexity to the communication between transmitters and receivers [1]. While the fifth generation (5G) system continue to be deployed globally, researchers have already begun their exploration of beyond 5G (B5G) and 6G technologies. 6G aims to achieve exceptional performance metrics, including ultra-high peak data rates (exceeding 1 Tbps), high reliability (over (>99.99%)), ultra-low latency (below 100μs), and remarkable energy efficiency [2]. Two promising technologies such as massive multiple input multiple outputs (MIMO) and millimeter wave (mmWave), are being considered to address the exponential growth of users while enhancing reliability and reducing latency. However, these advancements pose challenges due to their high hardware costs and energy consumption, necessitating the development of energy-efficient solutions for the future [3].

In the recent times, intelligent reflecting surfaces (IRS) have been proposed as an innovative technology to maintain uninterrupted connectivity and improve network coverage. IRS are planar structures that consists of large number of low cost passive elements. By adjusting the reflection property of the metamaterial elements, IRS can selectively amplify desired signals and attenuate undesired signals, thereby improving the overall signal quality at the receiver. However to achieve the interesting advantages of the IRS, accurate channel estimate techniques are needed. But, channel estimation in IRS is a challenging task since the IRS consists of passive elements, and during the channel assessment, the estimation of parameters becomes highly complex as it involves a huge number of channel parameters.

Several researchers have been working on channel estimation problem for IRS. The authors in [4], propose a layered pilot transmission scheme and a CANDECOMP/PARAFAC (CP) decomposition-based method for the joint estimation of the channels from multiple users without IRS. In [5], the authors propose two channel estimation scheme namely Khatri-Rao factorization and bilinear alternating least square (BALS) estimation. Most of the works focus on the cascaded channel estimation i.e., mobile station (MS)-IRS- base station (BS) link. However, it poses several challenges. One of the main challenges is, the required training time is proportional to the number of IRS units and hence large-scale IRS will result in extremely heavy training overhead. To address this issue, a new modified IRS namely hybrid intelligent reflecting metasurfaces (HIRS) is introduced [6] [7]. In HIRS, which combines both active (where the elements are connected to the digital processor via dedicated radio frequency (RF) chains which enables signal processing capabilities such as reception and decoding) and passive elements, channel estimation can be performed separately for the MS-IRS and IRS-BS links. This allows for point to point(p2p) channel estimation at both the IRS and the BS. In [7], the authors propose a low complexity tensor-based algorithm for p2p channel estimation for orthogonal frequency division multiplexing (OFDM)-MIMO systems. For narrowband channel estimation, the received signal at HIRS will be a third-order incomplete tensor, and for wideband, it is extended to fourth-order tensors.

All the works discussed above are limited to channel estimation for a single user only. In this paper, we consider HIRS for the narrowband channel in the case of a multiuser system. We decouple its channel estimation problem for multiuser HIRS scenario into MS to HIRS and from the HIRS to BS. To estimate the channel, we exploit the sparsity inherent in mm-Wave signals and formulate the received signal as third-order tensors. We then transform these incomplete tensors into complete observed tensors and recover the channel matrix using our proposed sparse BALS (sBALS) algorithm.

The rest of the paper is organised as follows, Section II presents the system model and problem formulation. Section III presents proposed narrowband channel estimation technique. Here we analyse the uniqueness condition for tensor completion and algorithm for recovering the channel matrix. Results of numerical studies are presented in Section IV. Section V draws some important conclusion and future directions.

Notations: Letters in lower case letters (a,b,c....), bold letters (**a,b,c....**), upper case bold letters (**A,B,C,....**) and upper case caligraphic letters ($\mathcal{A}, \mathcal{B}, \mathcal{C}$....) represents scalars, vectors, matrices, tensors respectively; $(.)^*$, $()^T$, $(.)^H$, $()^\dagger$ represent conjugate, transpose, conjugate transpose and pseudoinverse respectively; $\cdot_0, \cdot_2, \cdot_F$ denote norm-0,norm-2 and frobenius norm; $diag(.)$ represents diagonal matrix. $\odot$ is Khatri Rao product, $*$ is Hadamard product, $\otimes$ is Kronecker product; $C_n^2 n(n-1)/2$ and $C_2(A) \in C^{C_m^2 \times C_n^2}$ represents the 2^{nd} compound matrix comprising the determinants of all 2×2 submatrices.

II. System Model And Problem Formulation

The uplink channel model considers a multiuser scenario i.e., U mobile stations (MS) with a single antenna, HIRS with $N_I(N_x \times N_y)$ elements which are arranged in a two-dimensional structure, and a base station (BS) with multiple antennas. To facilitate channel estimation, specific metamaterial elements within the IRS are integrated with RF chains. Figure 1 illustrates the configuration of the wireless communication system utilizing IRS. The direct link between MS and the BS is assumed to be blocked.

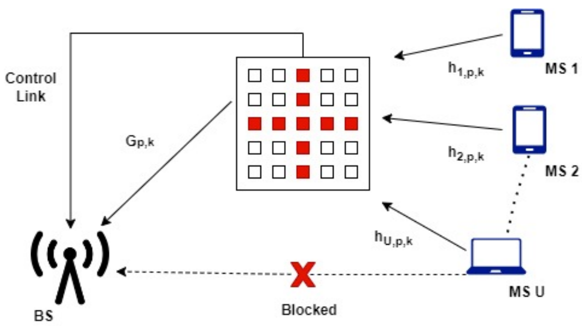

Fig. 1: MS-HIRS-BS uplink system [7]

All channels are assumed to be flat faded. The training length consist of P slots. Orthogonal pilot symbols are considered for transmission, which satisfies $x(i,p) * x(j,p) = 0$ if $i \neq j$ and $x(i,p) * x(j,p) = 1$ if $i = j$ ensuring orthogonality between symbols. The HIRS configuration of the size $(N_x \times N_y)$ is shown in Figure 2. The blue square boxes indicate effective 2x2 submatrices which can observe complete effective received signals.

The received training signal transmitted from MS to HIRS is given by,

$$\mathbf{y}_p = \sum_{u=1}^{U} \sum_{l=1}^{L} \mathbf{h}_{u,p,l}^{IM} x_{u,p} + \mathbf{n}_{u,p} \tag{1}$$

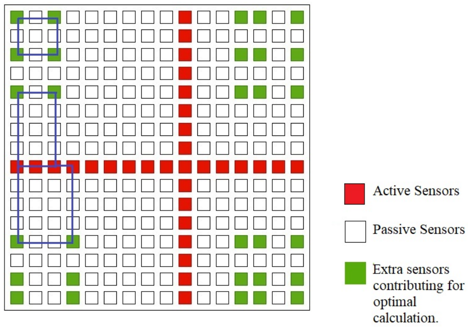

Fig. 2: Suboptimal IRS Configuration [7]

where L is the number of signal propagation paths, $\mathbf{h}_{u,p,l}^{IM}$ is the MS-IRS channel modelled by Saleh- Valenzula model [7], $x_{u,p}$ is the transmitted signal, $\mathbf{n}_{u,p}$ is the received signal noise. Alternatively, the above equation can be expressed as,

$$\mathbf{y}_p = \sum_{u=1}^{U} \sum_{l=1}^{L} (\beta_{u,l,p} \mathbf{a}_I(\phi_{u,l} \theta_{u,l})) x_{u,p} + \mathbf{n}_{u,p} \tag{2}$$

where $\beta_{u,l,p}$ represents path gain which will be different across different slots (P), $\mathbf{a}_I \mathbf{a}_{u,x,l} \otimes \mathbf{a}_{u,y,l}$ are steering vectors which is given by [3],

$$\mathbf{a}_{u,x,l} = \Big[1, e^{j2\pi \frac{d}{\lambda_c} sin\phi_{u,l} cos\theta_{u,l}}, e^{j2\pi \frac{d}{\lambda_c}(2) sin\phi_{u,l} cos\theta_{u,l}} \ldots \\ e^{j2\pi \frac{d}{\lambda_c}(N_x - 1) sin\phi_{u,l} cos\theta_{u,l}}\Big]^T \tag{3}$$

$$\mathbf{a}_{u,y,l} = \Big[1, e^{j2\pi \frac{d}{\lambda_c} sin\phi_{u,l} sin\theta_{u,l}}, e^{j2\pi \frac{d}{\lambda_c}(2) sin\phi_{u,l} sin\theta_{u,l}} \ldots \\ e^{j2\pi \frac{d}{\lambda_c}(N_y - 1) sin\phi_{u,l} sin\theta_{u,l}}\Big]^T \tag{4}$$

The angles of arrival (AoA) from u^{th} MS, namely $\theta_{u,l}$ and $\phi_{u,l}$ correspond to the azimuth and elevation angles. Here, the HIRS is assumed to be a uniform planar array (UPA). Let $\mathbf{g}_{u,\beta,l} = x_{u,p} \beta_{u,l,p}$ and, by concatenating the training length (P) and multipath, the factor matrices are given by, $\mathbf{A}_{x(y)} = \sum_{u=1}^{U} \mathbf{a}_{u,x(y)}$, $\mathbf{G}_\beta = \sum_{u=1}^{U} \mathbf{g}_{u,\beta}$. According to [8], the received signal can be modelled as tensor matricization which is given by,

$$\mathcal{Y} = \sum_{u=1}^{U} \sum_{l=1}^{L} \mathbf{a}_{u,x,l} \circ \mathbf{a}_{u,y,l} \circ \mathbf{g}_{u,\beta,l} + \mathcal{N} \tag{5}$$

where $\mathcal{Y} \in C^{N_x \times N_y \times P}$ and $\mathcal{N}$ is the received signal noise. The received signal at HIRS after detection by active elements is given by

$$\mathcal{Z} = \mathcal{W} * \mathcal{Y} \tag{6}$$

where $\mathcal{W} \in C^{N_x \times N_y \times P}$ represents the HIRS configuration with 0/1 represents passive/active entry state. The received signal in (6) is a third-order tensor that can be solved using CPD/PARAFAC model. Recovering $\mathcal{Z}$ is termed as tensor completion problem. It is noticed that *(i,j)*th entry of the passive element leads to $\mathcal{W}_{i,j,:} = 0_{P\times 1}$ which filters out the corresponding signal. Thus this problem can be termed fiber sampling tensor completion (FS-TC) problem. The $\mathcal{Z}$ tensor is reshaped to a matrix $\tilde{\mathbf{Z}} \in \mathbf{C}^{N_I \times P}$. The effective observed submatrix $\mathbf{Z}_{\text{sub}}$ is given by,

$$\mathbf{Z}_{\text{sub}} = \mathbf{S}_y \tilde{\mathbf{Z}} \tag{7}$$

Where $\mathbf{S}_y \in \{0,1\}^{N_A \times N_I}$ is a row extraction matrix that selects N_A active rows of $\tilde{\mathbf{Z}}$, that remain unaltered by $\mathcal{W}$.

III. Proposed PARAFAC Based Channel Estimation Technique

In this section, we present the proposed algorithm for channel recovery. In order to recover the channel parameters, we analyze some uniqueness condition for tensor completion [7]. These conditions play a crucial role in ensuring the effectiveness of the tensor completion process and achieving accurate channel recovery.

A. Uniqueness Condition for CPD Tensor

Before going to conditions, we represent the binary indicator $\mathbf{d_{sel}} \in \{0,1\}^{C^2_{N_x} C^2_{N_y}}$ which is given by [7],

$$\mathbf{d_{sel}} = [d_{(1,2)(1,2)}, d_{(1,2)(1,3)}, \dots d_{((N_x-1),(N_x))((N_y-1),(N_y))}] \tag{8}$$

where d is given by

$$d_{(i_1,i_2),(j_1,j_2)} = \begin{cases} 1, & \text{if } \mathcal{Y}_{i_1,j_1,:}, \mathcal{Y}_{i_1,j_2,:}, \mathcal{Y}_{i_2,j_1,:}, \mathcal{Y}_{i_2,j_2,:} \\ & \text{are observable fibers,} \\ 0, & \text{otherwise,} \end{cases}$$

and $1 \leq i_1 < i_2 \leq N_x$, $1 \leq j_1 < j_2 \leq N_y$. Physically $d_{(i_1,j_1)(i_2,j_2)}$ indicates whether the 2×2 submatrix of $\mathcal{Y}$ observes the complete effective signal, i.e., all elements in 2×2 submatrix should be 1 [7].

Defining the incomplete matrix $\mathbf{P}^{(l)}$ with indeterminate entries as,

$$p^{(l)}_{i,j} = \begin{cases} \mathbf{a}_{I(i-1)N_y+j}, & \text{if } \mathcal{W}_{i,j,:} = 1_{P\times 1}. \\ indeterminate, & \text{if } \mathcal{W}_{i,j,:} = 0_{P\times 1} \end{cases} \tag{9}$$

By utilizing graph theory, $\mathbf{P}^{(l)}$ can be mapped with bipartite graph denoted by $\mathcal{P}^{(l)}$. Now, $\tilde{\mathcal{P}}^{(l)}$ denotes the edge set associated with $\mathcal{P}^{(l)}$ where each edge (i,j) is associated with an edge weight $p^{(l)}_{i,j}$. Now the restricted bipartite graph denoted as $\hat{\mathcal{P}}^{(l)}$ which comprises the edge set from $\tilde{\mathcal{P}}^{(l)}$ where the corresponding edge weight $p^{(l)}_{i,j}$ is non-zero [7].

The uniqueness condition for tensor completion is given below,

Theorem 1 Consider a tensor with $\mathcal{Z} \in C^{N_x \times N_y \times P}$ with the factor matrices $\mathbf{A}_x, \mathbf{A}_y, \mathbf{G}_\beta$, then the conditions are given according to [8],

$$\begin{cases} rank(\mathbf{G}_\beta) = L \\ rank(diag(\mathbf{d}_{sel}))(C_2(\mathbf{A}_x) \odot C_2(\mathbf{A}_y))) = C^2_L \\ \hat{\mathcal{P}}^{(l)} \text{is a connected graph for every L} \end{cases} \tag{10}$$

If the factor matrices satisfy the uniqueness condition, the canonical polyadic decomposition (CPD) ensures unique normalization and permutation indeterminacy.

The first two cases in (10) will be satisfied if $P \geq L$ and $diag(\mathbf{d}_{sel})_0 \geq C^2_L$. The third condition can be satisfied if and only if $\mathbf{P}^{(l)}$ gets sampled at least once in all L cases. This condition ensures that the hybrid network, represented by $\mathcal{W}_{:,:,p}$, cannot divide the factor matrices $\mathbf{A}_x$ and $\mathbf{A}_y$ into disconnected matrices.

According to the uniqueness condition, it is possible to include multiple 2x2 submatrices as long as graph connectivity is maintained. In the fundamental configuration, the uniqueness condition is met by employing (N_x+N_y-1) active sensors to fully sample one row and one column. The minimum number of active sensors required to satisfy the uniqueness condition is discussed in [7] and can be expressed as, $N_A \geq N_x + N_y + L$ and $N_A \geq N_x + N_y + C^2_L - 1$.

B. PARAFAC decomposition

The factor matrices for PARAFAC decomposition is $\bar{\mathcal{Z}} = [[\mathbf{A}_x \mathbf{A}_y \mathbf{G}_\beta]]$. From [16], the PARAFAC decomposition using n-mode product notation is given as

$$\bar{\mathcal{Z}} = I_{3,N} \times_1 \mathbf{A}_x \times_2 \mathbf{A}_y \times_3 \mathbf{G}_\beta \tag{11}$$

By using trilinearity of PARAFAC decomposition, the above tensor can be unfolded into three forms as,

$$\bar{\mathbf{Z}}_1 = \mathbf{A}_x(\mathbf{G}_\beta \odot \mathbf{A}_y)^T \in C^{N_x \times N_y P} \tag{12}$$

$$\bar{\mathbf{Z}}_2 = \mathbf{A}_y(\mathbf{G}_\beta \odot \mathbf{A}_x)^T \in C^{N_y \times N_x P} \tag{13}$$

$$\bar{\mathbf{Z}}_3 = \mathbf{G}_\beta(\mathbf{A}_x \odot \mathbf{A}_y)^T \in C^{P \times N_x N_y} \tag{14}$$

The above unfolding of PARAFAC decomposition are the base to form bilinear alternating least square (BALS) channel estimation method [5].

C. Proposed Algorithm

It is assumed that the tensor rank is known (L=4). If the tensor rank is unknown, a minimum description length method can be adopted for estimating the rank [7]. The prerequisites for the algorithm is $\mathbf{Z}_{sub}$ and rank L. The overall idea is to interpret a tensor with missing fiber to full rank tensors [9].

To exploit the low-rank structure of the submatrix $\mathbf{Z}_{sub}$, we employ singular value decomposition (SVD) to compress its effective part. The SVD representation of $\mathbf{Z}_{sub}$ is then utilized, as follows [7]:

$$\mathbf{Z}_{sub} = \mathbf{U}_{sub} \sum\nolimits_{sub} \mathbf{V}_{sub}{}^{H} \tag{15}$$

where $\mathbf{U}_{sub} \in C^{N_A \times L}$, $\sum_{sub} \in C^{L \times L}$ and $\mathbf{V}_{sub} \in C^{L \times P}$. Defining $\bar{\mathbf{Z}}\mathbf{S}_y{}^T \mathbf{U}_{sub} \sum_{sub}$ and from $\bar{\mathbf{Z}}$, calculate $\bar{\mathcal{Z}} \in C^{N_x \times N_y \times P}$ by reshaping in tensor form. If range($\mathbf{Z}_{sub}$) = range ($\mathbf{U}_{sub} \sum_{sub}$), there exists a non-singular matrix $\mathbf{F} \in C^{L \times L}$ such that

$$\hat{\mathbf{G}}_\beta = \mathbf{V}^*_{sub} \mathbf{F} \tag{16}$$

The procedure is to convert the CPD with missing fibers to basic CPD that has only non-singular factor matrices. According to [8], let $\mathbf{Q}$ matrix is defined as $\mathbf{Q} = [\mathbf{Q}_1, 2\mathbf{Q}_2]$ where $\mathbf{Q}_1$ and $\mathbf{Q}_2$ be the effective parts of $\bar{\mathbf{Q}}_1$ and $\bar{\mathbf{Q}}_2$ indicated by $diag(\mathbf{d}_{sel})$

$$\bar{\mathbf{Q}}_1 = vec({C_2}^T(2 * \bar{\mathcal{Z}}(:,:,l) - 2 \cdot {C_2}^T(\bar{\mathcal{Z}}(:,:,l)) \tag{17}$$

$$\bar{\mathbf{Q}}_2 = vec(C_2^T(\bar{\mathcal{Z}}_{:,:,l1} + \bar{\mathcal{Z}}_{:,:,l2}) - C_2^T(\bar{\mathcal{Z}}_{:,:,l1}) - C_2^T(\bar{\mathcal{Z}}_{:,:,l2})) \tag{18}$$

Following the SVD decomposition of matrix $\mathbf{Q}$, a basis matrix $\mathbf{M}$ can be constructed using the L right singular vectors associated to the L smallest singular values of $\mathbf{Q}$. The columns of matrix $\mathbf{M}$ represent the basis of the kernel (null space) of $\mathbf{Q}$. Consequently, there exists a non-singular factor matrix $\mathbf{N}$ that satisfies

$$\mathbf{M} = (\mathbf{F}^{-T} \odot \mathbf{F}^{-T})\mathbf{N}^T \tag{19}$$

This corresponds to a matrix representation of the CPD. Given a matrix $\mathbf{M}$ of size $C^{L \times L \times L}$, we can represent it as a tensor $\mathcal{M}$ of the same size by applying a symmetrization mapping [7]. The CPD of $\mathbf{M}$ can be done with eigen value decomposition (EVD), thereby recovering $\mathbf{F}$. By substitute $\mathbf{F}$ in the equation (16), $\hat{\mathbf{G}}_\beta$ is estimated [7]. Based on the unfoldings (12-14), we propose modified BALS algorithm [5] named as sBALS algorithm for channel estimation which is specifically tailored for incomplete tensors. We aim to estimate the values of $\hat{\mathbf{A}}_x$ and $\hat{\mathbf{A}}_y$ by optimizing two separate cost functions,

$$\hat{\mathbf{A}}_x = \arg\min_{\mathbf{A_x}} \left\| \bar{\mathbf{Z}}_1 - \mathbf{A}_x (\hat{\mathbf{G}}_\beta \odot \mathbf{A}_y)^T \right\|_F^2 \tag{20}$$

$$\hat{\mathbf{A}}_y = \arg\min_{\mathbf{A_y}} \left\| \bar{\mathbf{Z}}_2 - \mathbf{A}_y (\hat{\mathbf{G}}_\beta \odot \mathbf{A}_x)^T \right\|_F^2 \tag{21}$$

Since we are dealing with an incomplete tensor, it is necessary to transform it into a full tensor. To achieve this, we start with the original tensor $\mathcal{Z}$ by replacing the missing fibers with the outer product of randomly initialized values of $\hat{\mathbf{A}}_x$ and $\hat{\mathbf{A}}_y$, and the estimated $\hat{\mathbf{G}}_\beta$ represented as $\mathcal{Z}_j^{new}$. This modified tensor allows us to perform matrix unfolding denoted by $\bar{\mathbf{Z}}^{new}_{j,1(2)}$ which is mapped to $\bar{\mathbf{Z}}_{1(2)}$, as described in equations (12-14). The solutions for $\hat{\mathbf{A}}_x$ and $\hat{\mathbf{A}}_y$ at j^{th} iteration can be obtained as follows,

$$\hat{\mathbf{A}}_{x(j,i)} = \bar{\mathbf{Z}}^{new}_{j,1} \left[\left(\hat{\mathbf{G}}_\beta \odot \hat{\mathbf{A}}_{y(j,i-1)} \right)^T \right]^\dagger \tag{22}$$

$$\hat{\mathbf{A}}_{y(j,i)} = \bar{\mathbf{Z}}^{new}_{j,2} \left[\left(\hat{\mathbf{G}}_\beta \odot \hat{\mathbf{A}}_{x(j,i)} \right)^T \right]^\dagger \tag{23}$$

The above steps are repeated until the convergence $\|e_2(i) - e_2(i-1) > \delta_2\|$ is accomplished, where $e_2(i) = \mathcal{Z}_j^{new}(i) - \mathcal{Z}_j^{new}(i-1)_F^2$ and δ_2 is the threshold parameter which denotes the reconstructed error at the i^{th} iteration. The tolerance level used here is $\delta_2 = 10^{-2}$. Update $\mathcal{Z}_j^{new}$ with $[\hat{\mathbf{A}}_{x(j)} \hat{\mathbf{A}}_{y(j)}, \hat{\mathbf{G}}_\beta]$. This process is repeated until the convergence of $\|e_1(j) - e_1(j-1) > \delta_1\|$ is satisfied, where $e_1(j) = \mathcal{Z}_j^{new} - \mathcal{Z}_{(j-1)}^{new}{}_F^2$ at j^{th} iteration. The tolerance level used is $\delta_1 = 10^{-3}$. At the end of convergence of e_1, we can get $\hat{A}_{x(y)}$=$\hat{A}_{x(y)(j)}$. The summary of modified fiber sampling tensor completion (FS-TC) with sBALS is given in algorithm 1.

Algorithm 1 Modified FS-TC for uplink multiuser narrowband system

1: Estimate $\hat{\mathbf{G}}_\beta$ from the equations (15-19)
2: Calculate $\mathcal{Z}_j^{new}$ by replacing zeros with random entries of factor matrices. Set j=0.
3: **while** $\|e_1(j) - e_1(j-1) > \delta_1\|$ **do**
4: Perform matrix unfolding denoted by $\bar{\mathbf{Z}}^{new}_{j,1(2)}$. Set i=0.
5: **while** $\|e_2(i) - e_2(i-1) > \delta_2\|$ **do**
6: Find the least square estimate of $A_x(j)$ and $A_y(j)$ as per the equation (22-23)
7: Repeat 3-4 until convergence
8: **end while**
9: Update the tensor with $\hat{\mathbf{A}}_{x(j)}$ & $\hat{\mathbf{A}}_{y(j)}$
10: Repeat 2 until convergence.
11: **end while**
12: Return the overall channel matrix $\hat{\mathcal{Y}} = \hat{\mathbf{A}}_x \circ \hat{\mathbf{A}}_y \circ \hat{\mathbf{G}}_\beta$.
13: Calculate NMSE for each user and calculate the overall average NMSE.

IV. Results And Discussion

In this section, we analyze the results of our simulations. We evaluate the performance of the channel estimator using the cumulative overall NMSE which is given by, NMSE = $\frac{\sum_{u=1}^{U} \mathcal{Y}_u - {y_u}_F^2}{\sum_{u=1}^{U} {\mathcal{Y}_u}_F^2}$. Moreover, the Angle of Arrival (AoA) $\phi_{u,l}, \theta_{u,l}$ is uniformly distributed in the range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, and $\beta_{u,l,p}$ follows a complex normal distribution $\mathcal{CN}(0,1)$. For further analysis and reference, the simulated parameters are tabulated in Table 1.

TABLE I: Simulated parameters

Parameters	Value
N_x	16
N_y	16
Carrier frequency (f_c)	28 GHz
Training length (P)	10
Propagation path (L)	4
Users	4

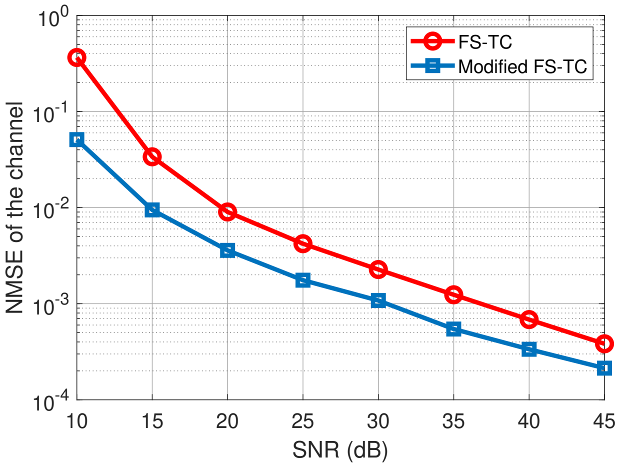

Fig. 3: NMSE of the estimated channel vs SNR

In Figure 3, the estimator performance is evaluated by plotting channel NMSE vs SNR. The results demonstrate that proposed algorithm exhibits consitently superior performance compared to the algorithm [7]. For instance, for achieving NMSE of 10^3, proposed algorithm requires only 30 dB whereas [7] requires around 35 dB.

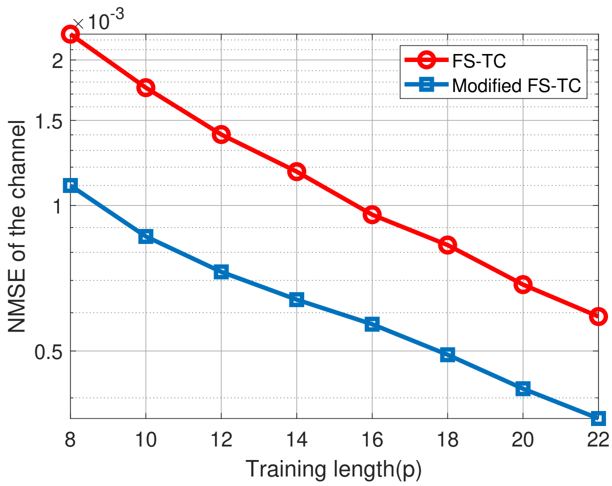

Fig. 4: NMSE of the estimated channel vs Training length

Figure 4 shows the channel performance on increasing the number of training length (P) for SNR = 30dB. From the figure, it shows that estimation achieves better accuracy on increasing P. Figure 5 shows channel the NMSE performance vs Missing ratio (MR) in percentage. MR is defined as the ratio of number of passive elements against total number of IRS elements (N_p/N_I). Based on this analysis, the proposed algorithm demonstrates its suitability for low-complexity IRS with a reduced number of active elements.

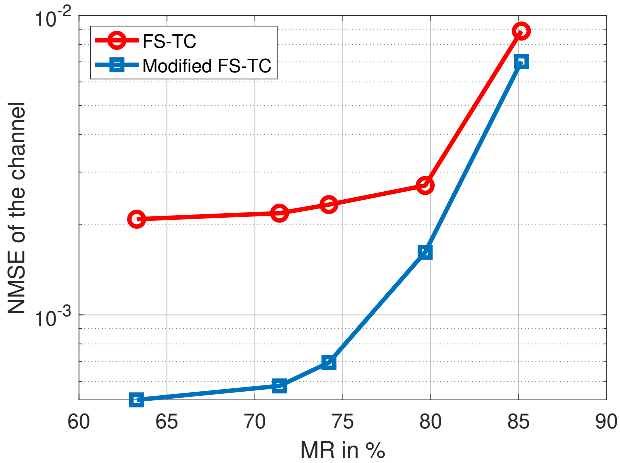

Fig. 5: NMSE of the estimated channel vs MR %

V. Conclusion

In our study, we conducted research on channel estimation in a narrowband multiuser system employing HIRS. The main focus is to overcome the challenges associated with channel estimation such as extremely training overhead in cascaded channel estimation through IRS. To address these challenges, we formulated the received training signal as a third-order incomplete CPD tensor and devised an algorithm for recovering the channel matrix. Our algorithm takes advantage of the HIRS layout and utilizes the inherent structure of the channel. Through simulation studies, we demonstrated that our proposed technique achieves significant improvement compared to the method presented in [7]. Overall, our work contributes to the advancement of channel estimation techniques in HIRS-assisted communication systems. It highlights the potential benefits of HIRS configurations and offers valuable insights for future research and development in this field.

VI. Acknowledgement

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