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Normalised Least Mean Fourth Algorithm for Hammerstein Spline Adaptive Filtering

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Abstract— **Hammerstein spline adaptive filtering (HSAF) is presented based on normalised version of least mean fourth (LMF) algorithm. HSAF comprises of a nonlinear memoryless adaptive lookup table with the spline interpolation function and linear adaptive filter modified by gradient-based scheme. LMF is determined by properties of the error term of the fourth power mean. Normalised version of LMF is applied on HSAF to get the fast convergence. Experimental results show that can provide the competitive results to the traditional least mean square algorithm.**

I. INTRODUCTION

Hammerstein spline adaptive filtering (HSAF) composes of a nonlinear memoryless adaptive lookup table (LUT) with the spline interpolation function and linear adaptive finite impulse response (FIR) filter [1]. In general, HSAF [1] and Wiener-HSAF [2] have been constructed on least mean square-based (LMS) algorithm that can manage well performance for the nonlinear system identification. Nonlinear Hammerstein function on SAF has been conducted against the impulsive noise environment for the nonlinear system identifications [1]- [4]. In [3], HSAF has been presented in terms of convergence properties with the good performance.

Consequently, the spline-based Hammerstein model [4] has been applied for the nonlinear digital cancellation in the full-duplex system. In [5], an adaptive normalised least mean square (NLMS) algorithm has been designed in the linear adaptive FIR filtering part of HSAF. In [6], HSAF with the diffusion scheme has been introduced for the distributed estimation.

According to the least mean fourth (LMF) algorithm, there are several approaches applied on the stochastic gradient-based algorithm to get the robust and stability performance [7]- [9]. In [8], LMF cost function has been modified on the adaptive affine projection algorithm for the system identification. Analysis of normalised least mean fourth (NLMF) has been investigated to protect the degradation for the adaptive noise cancellation in [9].

The main objective of this paper is to derive the adaptive NLMF algorithm for HSAF model. Firstly, HSAF model is described in brief in Section II. Then, proposed NLMF algorithm for HSAF model will be explained with the adaptive averaging step-size method in Section III. Experimental results will be shown in Section IV and conclusion will be concluded in Section V.

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II. HSAF MODEL

Following [1], the HSAF network shown in Fig. 1 that incorporates with the non-linear and linear parts. Nonlinear part is applied for the Hammerstein-based nonlinear identification systems [10] with an adaptive LUT with the spline interpolation function. In which, the linear part is an adaptive FIR filtering that consists of three basic elements as the unit-delay element, multiplier and adder. Delay elements in Fig. 1 are identified by the unit-delay operator z^{-1}.

Regard to an estimated error $e(n)$ as

$$e_k = d_k - y_k \ , \tag{1}$$

$$y_k = \mathbf{w}_k^T \, \mathbf{s}_k \ , \tag{2}$$

where y_k is the HSAF output, d_k is the desired signal and $\mathbf{w}_k$ is the linear tap-weight coefficient. And $\mathbf{x}_k$ is the input vector with the length of tap delay M as

$$\mathbf{w}_k = [\ w_{0,k} \ w_{1,k} \ \ldots \ w_{M-1,k} \ ] \ , \tag{3}$$

$$\mathbf{x}_k = [\ x_k \ x_{k-1} \ \ldots \ x_{k-M+1} \ ] \ . \tag{4}$$

The LUT output $\mathbf{s}_k$ is given by [11]

$$\mathbf{s}_k = \varphi(u_k) = \mathbf{u}_k^T \, \mathbf{C} \, \mathbf{q}_{i,k} \ , \tag{5}$$

$$\mathbf{u}_k = [u_k^3, \ u_k^2, \ u_k, \ 1]^T \ , \tag{6}$$

where $\mathbf{q}_{i_k}$ is the adaptive control points tap-weight coefficient as

$$\mathbf{q}_{i,k} = [\ q_{i,k} \ q_{i+1,k} \ q_{i+2,k} \ q_{i+3,k} \ ]^T \ . \tag{7}$$

Local parameter u_k and span index i can be evaluated as [1]

$$u_k = \frac{x_k}{\Delta x} - \left\lfloor \frac{x_k}{\Delta x} \right\rfloor \ , \tag{8}$$

$$i = \left\lfloor \frac{x_k}{\Delta x} \right\rfloor + \frac{Q-1}{2} \ , \tag{9}$$

where Δx is the uniform space between two connected adaptive control points coefficient. Q is the number of control points coefficient, and $\lfloor \cdot \rfloor$ is floor operator.

The *B-spline* matrix is used for $\mathbf{C}$ as [11]

$$\mathbf{C} = \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \ . \tag{10}$$

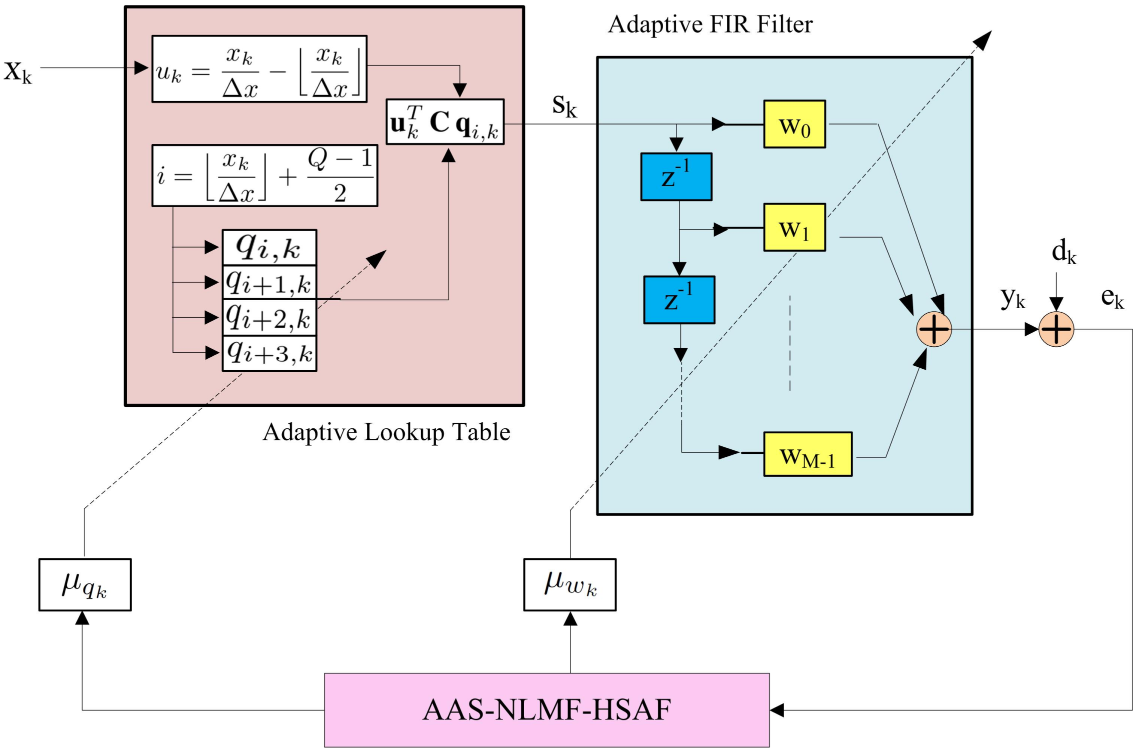

Fig. 1. Nonlinear-linear network of HSAF based on AAS-NLMF algorithm.

It is seen that the LUT output s_k is correlated with the local parameter u_k and index i of control points tap-weight coefficient.

III. Proposed AAS-NLMF-HSAF Algorithm

In this section, the proposed NLMF algorithm with adaptive averaging step-size method for HSAF (AAS-NLMF-HSAF) are considered in details.

Following [8] and [12], the cost function based on LMF algorithm for minimising constraint can be determined as

$$\min_{\mathbf{w}_k} \nu_k = \frac{1}{2}(\mathbf{u}_k^T\,\mathbf{u}_k)^{-1}\,\xi^4 \;, \tag{11}$$

$$\text{subject to} \quad \|\hat{\mathbf{w}}_k - \hat{\mathbf{w}}_{k-1}\|_2^2 \;\leq\; \delta^2 \;, \qquad \|\hat{\mathbf{q}}_{i,k} - \hat{\mathbf{q}}_{k-1}\|_2^2 \;\leq\; \delta^2 \tag{12}$$

where δ denotes the small constant. An estimated error ξ_k is given by

$$\xi_k = d_k - \hat{\mathbf{w}}_{k-1}^T\,\mathbf{s}_k \;. \tag{13}$$

Then, the proposed adaptive tap-weight coefficient $\hat{\mathbf{w}}_k$ and adaptive control points tap-weight coefficient $\hat{\mathbf{q}}_{i,k}$ can be obtained as

$$\hat{\mathbf{w}}_k = \hat{\mathbf{w}}_{k-1} - \mu_{w_k}\,\frac{\partial \mathbb{J}(\hat{\mathbf{w}}_k, \hat{\mathbf{q}}_{i,k})}{\partial \hat{\mathbf{w}}_k} \;, \tag{14}$$

$$\hat{\mathbf{q}}_{i,k} = \hat{\mathbf{q}}_{i,k-1} - \mu_{q_{i,k}}\,\frac{\partial \mathbb{J}(\hat{\mathbf{w}}_k, \hat{\mathbf{q}}_{i,k})}{\partial \hat{\mathbf{q}}_{i,k}} \;, \tag{15}$$

where μ_{w_k} and $\mu_{q_{i,k}}$ are the step-size parameters of tap-weight coefficients $\hat{\mathbf{w}}_k$ and $\hat{\mathbf{q}}_{i,k}$, respectively.

Differentiating in (11) by chain rule [11] with respect to $\hat{\mathbf{w}}_k$, we get

$$\frac{\partial \mathbb{J}(\hat{\mathbf{w}}_k, \hat{\mathbf{q}}_{i,k})}{\partial \hat{\mathbf{w}}_k} = -2\,(\mathbf{u}_k^T\,\mathbf{u}_k)^{-1}\left\{\mathbf{s}_k\,\xi_k^3\right\} \;. \tag{16}$$

In that way by considering the gradient of (11) with respect to $\hat{\mathbf{q}}_{i,k}$, we arrive at

$$\frac{\partial \mathbb{J}(\hat{\mathbf{w}}_k, \hat{\mathbf{q}}_{i,k})}{\partial \hat{\mathbf{q}}_{i,k}} = -2\,(\mathbf{u}_k^T\,\mathbf{u}_k)^{-1}\left\{\mathbf{u}_k\,\mathbf{C}^T\,\hat{\mathbf{w}}_k\,\xi_k^3\right\} \;. \tag{17}$$

By substituting (16) into (14), the proposed $\hat{\mathbf{w}}_k$ can be furnished by

$$\therefore\, \hat{\mathbf{w}}_k = \hat{\mathbf{w}}_{k-1} + \frac{\mu_{w_k}}{\mathbf{u}_k^T\,\mathbf{u}_k}\left\{\mathbf{s}_k\,\xi_k^3\right\} \;, \tag{18}$$

Then, substituting (17) into (15), the proposed $\hat{\mathbf{q}}_{i,k}$ can be illustrated by

$$\therefore\, \hat{\mathbf{q}}_{i,k} = \hat{\mathbf{q}}_{i,k-1} + \frac{\mu_{q_{i,k}}}{\mathbf{u}_k^T\,\mathbf{u}_k}\left\{\mathbf{u}_k\,\mathbf{C}^T\,\hat{\mathbf{w}}_k\,\xi_k^3\right\} \;, \tag{19}$$

A. Adaptive Averaging Step-size Algorithms

In order to accelerate the convergence rate, an adaptive averaging step-size method μ_{w_k} of $\hat{\mathbf{w}}_k$ is implemented with the help of square of estimated error ξ_k [13] as

$$\mu_{w_k} = \beta_w\,\mu_{w_{k-1}} + \delta_w\,\zeta_k^2 \;, \tag{20}$$

$$\zeta_k \;= \rho\,\zeta_{k-1} + (1-\rho)\,\xi_k^2 \;. \tag{21}$$

where δ_w denotes a scaling parameter, $0 < \beta_w < 1$ and ρ is close to 1.

Therefore, an adaptive averaging step-size μ_{q_k} scheme of $\hat{\mathbf{q}}_{i,k}$ is defined by

$$\mu_{q_k} = \beta_q\,\mu_{q_{k-1}} + \delta_q\,\xi_k^2 \;, \tag{22}$$

where $\delta_q > 0$ and $0 < \beta_q < 1$.

TABLE I
SUMMARY OF PROPOSED AAS-NLMF-HSAF ALGORITHM.

For $k = 1, 2, \dots,\ \mathbb{N}$

1) To arrange the local parameter $\mathbf{u}_k$ as:

$$\mathbf{u}_k = [u_k^3,\ u_k^2,\ u_k,\ 1]^T$$
$$u_k = \frac{x_k}{\Delta x} - \left\lfloor \frac{x_k}{\Delta x} \right\rfloor$$

2) To organise index i as:

$$i = \left\lfloor \frac{x_k}{\Delta x} \right\rfloor + \frac{Q-1}{2}$$

3) To compute ξ_k as:

$$\xi_k = d_k - \hat{\mathbf{w}}_{k-1}^T \mathbf{s}_k$$
$$\mathbf{s}_k = \mathbf{u}_k^T \mathbf{C} \mathbf{q}_{i,k}$$

4) To determine μ_{w_k}

$$\mu_{w_k} = \beta_w \mu_{w_{k-1}} + \delta_w \zeta_k^2$$
$$\zeta_k = \rho \zeta_{k-1} + (1-\rho) \xi_k^2$$

5) To compute μ_{q_k}

$$\mu_{q_k} = \beta_q \mu_{q_{k-1}} + \delta_q \xi_k^2$$

6) To calculate $\hat{\mathbf{w}}_k$ and $\hat{\mathbf{q}}_{i,k}$

$$\hat{\mathbf{w}}_k = \hat{\mathbf{w}}_{k-1} + \frac{\mu_{w_k}}{\mathbf{u}_k^T \mathbf{u}_k} \left\{ \mathbf{s}_k \xi_k^3 \right\}$$
$$\hat{\mathbf{q}}_{i,k} = \hat{\mathbf{q}}_{i,k-1} + \frac{\mu_{q_{i,k}}}{\mathbf{u}_k^T \mathbf{u}_k} \left\{ \mathbf{u}_k \mathbf{C}^T \hat{\mathbf{w}}_k \xi_k^3 \right\}$$

end

IV. SIMULATION RESULTS

Random processes are used for the computer simulations. In the Wiener system, performance of proposed NLMF-HSAF algorithm is compared with the traditional LMS-HSAF [1] over 200 Monte-Carlo trials and 2,000 samples. The input signal x_k is circulated by

$$x_k = \gamma \cdot x_{k-1} + \sqrt{1-\gamma^2} \varrho_k\ , \tag{23}$$

where ϱ_k is a zero mean and unitary variance of white Gaussian noise. Parameter γ is set to $[0.01, 0.99]$.

The simulation is made up of an unknown Wiener system by [14]

$$\mathbf{w}_0 = [0.6,\ -0.4,\ 0.25,\ -0.15,\ 0.1]\ .$$

Length of 23 points LUT $\mathbf{q}_0$ function is orchestrated by an interpolation as [15]- [16]

$$\mathbf{q}_0 = \{-2.2, -2, -1.8, \dots, -1.0, -0.8, -0.91, 0.42, -0.01, -0.1, 0.1, -0.15, 0.58, 1.2, 1.0, 1.2, \dots, 2.0, 2.2\}$$

Parameters are used for all algorithms as $\Delta x = 0.2$, the third degree spline $Q = 3$ and number of tap-weight coefficients $M = 7$. Initial parameters of proposed AAS-NLMF-HSAF shown in Table I for adaptive control points vector $\hat{\mathbf{q}}_{i,k}$ and adaptive FIR filter $\hat{\mathbf{w}}_k$ are as follows: $\mu_q(0) = \mu_w(0) = 7.5 \times 10^{-3}$, $\beta_q = 1 \times 10^{-3}$, $\delta_q = 1 \times 10^{-3}$, $\beta_w = 1 \times 10^{-3}$, $\delta_w = 1 \times 10^{-3}$, $\rho = 0.975$, a signal to noise ratio $SNR = 10, 20$ dB.

TABLE II
SUMMARY OF PROPOSED LMS-HSAF ALGORITHM [1].

For $k = 1, 2, \dots,\ \mathbb{N}$

1) To arrange the local parameter $\mathbf{u}_k$ as:

$$\mathbf{u}_k = [u_k^3,\ u_k^2,\ u_k,\ 1]^T$$
$$u_k = \frac{x_k}{\Delta x} - \left\lfloor \frac{x_k}{\Delta x} \right\rfloor$$

2) To organise index i as:

$$i = \left\lfloor \frac{x_k}{\Delta x} \right\rfloor + \frac{Q-1}{2}$$

3) To compute e_k as:

$$e_k = d_k - \mathbf{w}_{k-1}^T \mathbf{s}_k$$
$$\mathbf{s}_k = \mathbf{u}_k^T \mathbf{C} \mathbf{q}_{i,k}$$

4) To calculate $\mathbf{w}_k$ and $\mathbf{q}_{i,k}$

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu_w \mathbf{s}_k e_k$$
$$\mathbf{q}_{i,k} = \mathbf{q}_{i,k-1} + \mu_q \mathbf{u}_k \mathbf{C}^T \mathbf{w}_k e_k$$

end

Initial parameters of LMS algorithm for HSAF model (LMS-HSAF) shown in Table II for adaptive control points vector $\mathbf{q}_{i,k}$ and adaptive FIR filter $\mathbf{w}_k$ are as follows: $\delta_w = 1 \times 10^{-3}$, the fixed step-size parameters $\mu_w = \mu_q = 7.5 \times 10^{-3}$ and $SNR = 10, 20$ dB.

Fig. 2 depicts the curves of various step-size μ_{q_k} of $\hat{\mathbf{q}}_{i,k}$ and Fig. 3 depicts the curves of step-size μ_{w_k} of $\hat{\mathbf{w}}_k$. It is concluded that the proposed adaptive averaging step-size algorithm for both $\hat{\mathbf{w}}_k$ and $\hat{\mathbf{q}}_{i,k}$ are shown to converge to their own values.

Comparison of mean square error (MSE) of proposed algorithms with $\gamma = 0.10, 0.75$ in (23) are shown in Fig. 4. MSE trend of proposed NLMF-HSAF and traditional LMS-HSAF [1] are depicted with the different γ and SNR=20dB, where the MSE is computed as $10 \log(\xi_k^2)$ in dB. It is seen that the MSE curves of proposed HSAF-NLMS algorithm can implement well performance in comparison with the LMS-SAF algorithm at the steady state.

V. CONCLUSIONS

In this paper, the proposed normalised least mean fourth algorithm based on Hammerstein spline adaptive filtering (NLMF-HSAF) structure has been orchestrated with the minimising constraint cost function. The proposed adaptive nonlinear tap-weight control points coefficient and linear tap-weight vector have been described to derive with the help of chain rule. In which, an adaptive averaging step-size schemes for nonlinear and linear adaptive filters are applied

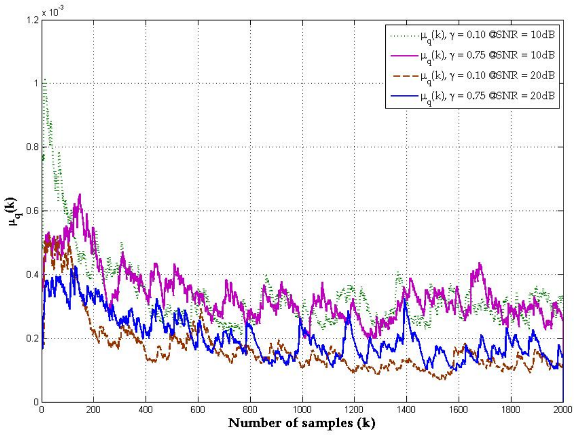

Fig. 2. Step-size curves μ_{q_k} of $\hat{\mathbf{q}}_{i,k}$ with $\gamma = 0.10, 0.75$ and $SNR = 10, 20$ dB.

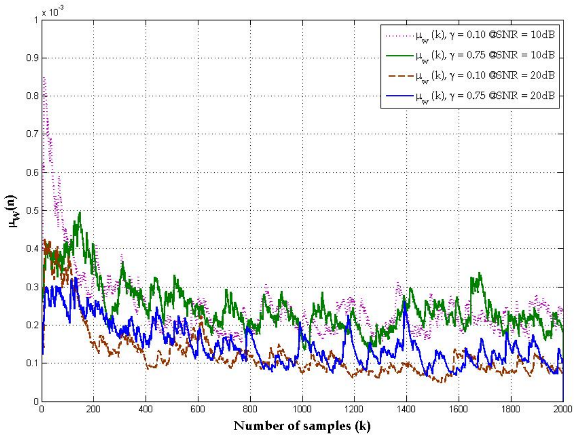

Fig. 3. Step-size curves μ_{w_k} of $\hat{\mathbf{w}}_k$ with $\gamma = 0.10, 0.75$ and $SNR = 10, 20$ dB.

to ensure on the choice of fast convergence with low complexity. In short, proposed NLMF-HSAF algorithm can conduct the good performance compared with the traditional LMS-HSAF algorithm.

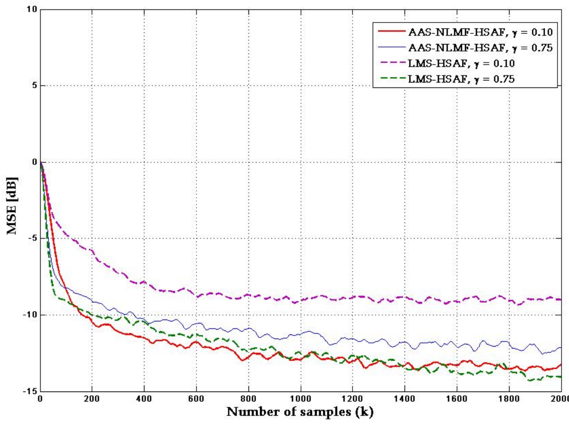

Fig. 4. MSE trends of proposed NLMF-HSAF and LMS-HSAF [1] with $SNR = 20dB$, where $\gamma = 0.10, 0.75$.

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