IRS Assisted FBMC Waveform: Channel Estimation and Reflecting Coefficients Optimization

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Abstract— **In this paper, single-input single-output filter bank multicarrier (FBMC) waveform based on offset quadrature amplitude modulation (OQAM) is investigated in conjunction with intelligent reflecting surface (IRS). A frame structure for IRS assisted FBMC waveform is designed for channel frequency response (CFR) estimation, followed by IRS reflecting coefficients optimization. In particular, an ON/OFF channel estimation technique is proposed by inserting guard symbols between the adjacent training symbols. The guard symbols are utilized to mitigate the inter-symbol-interference between the adjacent training symbols, and to help in calculating the inherent intrinsic interference in FBMC waveform. We next investigate a random phase initialization based successive convex approximation technique to jointly optimize the IRS reflecting coefficients and sub-carriers transmit power allocation using both perfect and imperfect CFRs. Our simulation results demonstrate the accuracy of proposed CFR estimation and reflecting coefficient optimization schemes, and the effect of the guard symbols on their performances.**

I. Introduction

Recently, intelligent reflecting surface (IRS) has evolved as a promising technology, which enables changeable and smart channels for beyond 5G (B5G) wireless systems. The passive reflecting elements of IRS can independently induce a controllable amplitude and/or phase in signal incident on them, thereby reconfiguring wireless channels between the transmitter and receiver to get desired propagation characteristics. Orthogonal frequency division multiplexing (OFDM) has been used with IRS to avoid inter-symbol-interference (ISI) over frequency selective channels [1], [2]. However, OFDM waveform is highly sensitive to practical impairments such as carrier frequency offset (CFO) and time offset (TO). This happens because OFDM uses rectangular pulse shaping filters for each sub-carrier, due to which the amount of out-of-band (OOB) radiation is significantly higher. As a result, OFDM systems suffer from severe inter-carrier-interference (ICI) in the presence of CFO and/or TO. In light of the aforementioned drawbacks of OFDM, filter bank multicarrier based on offset quadrature amplitude modulation (FBMC/OQAM) has emerged as a potential alternative signalling technique. Due to well frequency-time (FT) localized filters in FBMC/OQAM waveform, OOB emission is significantly lower than its OFDM counterpart. This makes FBMC/OQAM waveform robust against errors arising due to CFO and TO.

In an IRS assisted wireless system, it is critical to design IRS reflecting coefficients (i.e., amplitude's and phase shifts) in order to constructively superpose the reflected signals from IRS with the signals from other paths at the receiver in a best possible way. This demands an accurate channel estimation between the user(s) and the base station (BS) through IRS, followed by an efficient optimization of the reflecting coefficients. For example, the works in [3]–[8] focused on frequency-flat fading channels for narrow-band wireless communications, and for constructive superposition, reflecting coefficients of IRS are designed to align phase of the user-BS direct link with that of the user-IRS-BS reflected link.

Recently, the performance of FBMC/OQAM waveform has been studied with 5G technologies such as massive multiple input multiple output (MIMO) [9] and millimeter wave (mmWave) [10]. To the best of our knowledge, no work has been published in context of IRS assisted FBMC systems. In contrast to OFDM, the adjacent time domain FBMC/OQAM symbol overlap with each other. Due to this, it is challenging to design IRS assisted FBMC/OQAM wireless systems. For example, channel estimation in IRS-FBMC waveform requires insertion of zeros between the adjacent training symbols to suppress the overlapping between them. In addition, one also has to compute the intrinsic interference at the receiver before channel estimation. Further, the transmit power allocation over FBMC/OQAM sub-carriers and the IRS reflecting coefficients are coupled, and hence a joint optimization of both is required to maximize the user's achievable spectral efficiency. Also, the scale of the joint optimization issue grows with the number of IRS elements and/or FBMC/OQAM sub-carriers. However, to reap the advantages of both IRS and FBMC technologies, it is imperative to research on amalgamation of IRS and FBMC/OQAM systems. In view of the above observations, the key contributions of this work are bulleted below.

- We begin by deriving system model for IRS assisted FBMC/OQAM waveform over frequency selective channels. Subsequently, a frame structure is designed for channel estimation and reflecting coefficients optimization for IRS-FBMC waveform. As shown later in section III, the proposed frame structure is radically different from its OFDM counterpart due to the overlapping nature of time-domain FBMC symbols.
- By inserting zeros between the adjacent time-domain FBMC training symbols to avoid the overlapping between them, an ON/OFF scheme is investigated for user-IRS-BS channel estimation.
- An optimization scheme is investigated to jointly optimize IRS reflecting coefficients and transmit power allocation to FBMC/OQAM sub-carriers.

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- Numerical examples are presented to demonstrate the accuracy of the proposed schemes, and the effect of the number of zeros on their channel estimation mean-squared-error and achievable spectral efficiency.

II. IRS-FBMC Waveform System Model

We consider an uplink IRS aided FBMC waveform, wherein a single user with single antenna communicates to the BS through IRS, and through a direct link. It is assumed that the IRS surface comprises of R passive reflecting elements which are controlled by an IRS controller. For desired reflection, IRS reflecting coefficients are adjusted by the IRS controller. In order to compute the optimal reflecting coefficients and to decode user information, the BS needs to estimate the direct and reflected channels using training symbols.

The discrete time equivalent baseband signal transmitted by the user using SISO-FBMC/OQAM waveform is given by [11]

$$s[k]=\sum_{m=0}^{N-1}\sum_{n\in Z}d_{m,n}\chi_{m,n}[k], \tag{1}$$

where k is the sample index, $d_{m,n}$ denotes a real valued OQAM symbol at time index n and sub-carrier index m and N is the number of sub-carriers. The FBMC basis function

$$\chi_{m,n}[k]=p[k-nM]\,e^{j\frac{2\pi}{N}mk}e^{j\varphi_{m,n}}, \tag{2}$$

where $M=N/2$ and $p[k]$ is the discrete-time symmetrical real valued pulse of length N_p which is different from the rectangular pulse in OFDM waveform. The phase factor $\varphi_{m,n}=(\pi/2)(m+n)-(\pi mn)$ is defined as modulo π [11]. The real valued OQAM symbols $d_{m,n}$, each of duration $T/2$, are drawn by extracting the imaginary and real parts of complex QAM symbols $b_{m,n}$ with an offset of $T/2$, where T is the complex QAM symbol duration [12]. The real valued OQAM symbols $d_{m,n}$ are assumed to be temporally independent and identically distributed (i.i.d.) with power p_m such that $E\left[d_{m,n}(d_{m,n})^*\right]=p_m$. To recover the symbols $d_{m,n}$, the basis function $\chi_{m,n}[k]$ satisfies the following orthogonality in the real field [11]

$$\Re\left\{\sum_{k=-\infty}^{\infty}\chi_{m,n}[k]\,\chi^*_{\bar{m},n}[k]\right\}=\delta_{m,\bar{m}}\delta_{n,\bar{n}}, \tag{3}$$

where $\delta_{m,n}$ denotes the Kronecker delta. For ease of mathematical analysis, let $\xi^{\bar{m},\bar{n}}_{m,n}=\sum_{k=-\infty}^{\infty}\chi_{m,n}[k]\,\chi^*_{\bar{m},n}[k]$, where

$$\xi^{\bar{m},\bar{n}}_{m,n}=\begin{cases}1, & \text{if}\,(m,n)=(\bar{m},\bar{n})\\ \text{Imaginary}, & \text{if}\,(m,n)\neq(\bar{m},\bar{n}).\end{cases} \tag{4}$$

Let $\tilde{\boldsymbol{h}}_d=\left[\tilde{h}_0,\cdots,\tilde{h}_{L_0-1},\mathbf{0}_{1\times(N-L_0)}\right]^T\in\mathbb{C}^{N\times1}$ be a zero-padded L_0-tap user to BS (direct link) baseband channel, where $\mathbf{0}_{1\times N}$ represents a zero vector of size $1\times N$. Let $\tilde{\boldsymbol{h}}_{1_i}\in\mathbb{C}^{L_1\times1}$ denote L_1-tap IRS to BS channel and $\tilde{\boldsymbol{h}}_{2_i}\in\mathbb{C}^{L_2\times1}$ is an L_2-tap user to IRS channel for the i-th IRS reflecting element. Each IRS element reflects the signal incident on it by introducing an amplitude and phase to it. Let $\phi_i=\beta_i e^{j\theta_i}$ be the reflecting coefficient of the i-th IRS element where the amplitude coefficient $\beta_i\in[0,1]$ and the phase shift $\theta_i\in[-\pi,\pi)$. This implies that $|\phi_i|\leq1$. Let $\boldsymbol{\phi}=[\phi_1,\cdots,\phi_R]^T\in\mathbb{C}^{R\times1}$ denote a vector of R IRS coefficients. The composite channel from user to BS through i-th IRS element (user-IRS-BS) can be written as $\tilde{\boldsymbol{h}}_{1_i}*\phi_i*\tilde{\boldsymbol{h}}_{2_i}=\phi_i\left(\tilde{\boldsymbol{h}}_{1_i}*\tilde{\boldsymbol{h}}_{2_i}\right)\in\mathbb{C}^{L_r\times1}$, where $L_r=L_1+L_2-1$ and $*$ denote the convolution operation. Let the matrix $\tilde{\mathbf{H}}_r=\left[\tilde{\boldsymbol{h}}_{c_1}\quad\tilde{\boldsymbol{h}}_{c_2}\quad\cdots\quad\tilde{\boldsymbol{h}}_{c_R}\right]\in\mathbb{C}^{N\times R}$ where $\tilde{\boldsymbol{h}}_{c_i}=\left[\left(\tilde{\boldsymbol{h}}_{1_i}*\tilde{\boldsymbol{h}}_{2_i}\right)^T,\mathbf{0}_{1\times(N-L_r)}\right]^T\in\mathbb{C}^{N\times1}$ gives the zero-padded concatenated user to BS channel through the i-th IRS element. The composite user-IRS-BS channel for all the IRS elements can be expressed as

$$\tilde{\boldsymbol{h}}_r=\tilde{\mathbf{H}}_r\boldsymbol{\phi}. \tag{5}$$

Now, the combined channel impulse response (CIR) between the user and the BS can be written as the superposition of user-BS direct channel and user-IRS-BS reflected channel as

$$\tilde{\boldsymbol{h}}=\tilde{\boldsymbol{h}}_d+\tilde{\boldsymbol{h}}_r=\tilde{\boldsymbol{h}}_d+\tilde{\mathbf{H}}_r\boldsymbol{\phi}=\tilde{\boldsymbol{h}}_d+\sum_{i=1}^{R}\tilde{\boldsymbol{h}}_{c_i}\phi_i. \tag{6}$$

It can be noted that the knowledge of $\tilde{\boldsymbol{h}}_{c_i}$ for all the IRS elements and $\boldsymbol{\phi}$ is sufficient to represent the user-IRS-BS channel. One does not need the explicit knowledge of $\tilde{\boldsymbol{h}}_{1_i}$ and $\tilde{\boldsymbol{h}}_{2_i}$. The FBMC signal received at the BS is given as

$$y[k]=s[k]*h[k]+\eta[k], \tag{7}$$

where $\eta[k]$ is temporally white noise which is distributed as $\mathcal{CN}(0,\sigma^2_\eta)$ and $h[k]$ is the k-th element of the channel vector $\tilde{\boldsymbol{h}}$. The FBMC demodulated signal at sub-carrier index $\bar{m}$ and symbol time index $\bar{n}$ can be obtained as $y_{\bar{m},\bar{n}}=\sum_{k=-\infty}^{+\infty}y[k]\chi^*_{\bar{m},\bar{n}}[k]$ [11]. Substituting (1), (2) and (7), $y_{\bar{m},\bar{n}}$ can be written as in (8) (at the top of next page), where $\eta_{\bar{m},\bar{n}}=\sum_{k=-\infty}^{+\infty}\eta[k]\chi^*_{\bar{m},\bar{n}}$ is the demodulated noise at the BS.

The duration of pulse $p[k]$ in FBMC is typically integer multiples of T. For example, reference [11] has taken it to be $4T$. Under the commonly used assumption that the symbol time is sufficiently longer than the maximum channel delay spread, the impulse response $p[k]$ is almost constant over the CIR length. Hence the following condition holds true [11]

$$p[k-l-nM]\approx p[k-nM]. \tag{9}$$

Substituting (4) and (9) in (8), we get

$$y_{\bar{m},\bar{n}}=\sum_{m=0}^{N-1}\sum_{n\in\mathbb{Z}}d_{m,n}\sum_{l=0}^{N-1}h[l]e^{-j\frac{2\pi}{N}ml}\xi^{\bar{m},\bar{n}}_{m,n}+\eta_{\bar{m},\bar{n}}. \tag{10}$$

Now substituting (6) in (10), we get

$$y_{\bar{m},\bar{n}}=\sum_{m=0}^{N-1}\sum_{n\in\mathbb{Z}}d_{m,n}\left(H_{d_m}+\sum_{i=1}^{R}H_{c_{i_m}}\phi_i\right)\xi^{\bar{m},\bar{n}}_{m,n}+\eta_{\bar{m},\bar{n}}, \tag{11}$$

where $H_{d_m}=\sum_{l=0}^{N-1}\tilde{h}_d[l]\,e^{-j\frac{2\pi}{N}ml}$ and $H_{c_{i_m}}=\sum_{l=0}^{N-1}\tilde{h}_{c_i}[l]\,e^{-j\frac{2\pi}{N}ml}$ are the CFRs of the direct and concatenated channels at m-th sub-carrier index and $\tilde{h}_d[l]$ and $\tilde{h}_{c_i}[l]$ are the l-th component of the vectors $\tilde{\boldsymbol{h}}_d$ and $\tilde{\boldsymbol{h}}_{c_i}$ respectively. Let $\boldsymbol{h}_d=\left[H_{d_0}\cdots H_{d_{N-1}}\right]^T\in\mathbb{C}^{N\times1}$ and $\boldsymbol{h}_{c_i}=\left[H_{c_{i_0}}\cdots H_{c_{i_{N-1}}}\right]^T\in\mathbb{C}^{N\times1}$ be the CFR vectors

$$y_{\bar{m},\bar{n}} = \sum_{m=0}^{N-1}\sum_{n\in\mathbb{Z}} d_{m,n} \sum_{l=0}^{N-1} h[l]e^{-j\frac{2\pi}{N}ml} \sum_{k=-\infty}^{+\infty} p[k-l-nM]p[k-\bar{n}M]e^{j(\varphi_{m,n}-\varphi_{\bar{m},\bar{n}})}e^{j\frac{2\pi}{N}(m-\bar{m})k} + \eta_{\bar{m},\bar{n}} \quad (8)$$

for the direct and reflected links, respectively. Then, the matrix $\mathbf{H}_r = [\boldsymbol{h}_{c_1} \cdots \boldsymbol{h}_{c_R}] \in \mathbb{C}^{N\times R}$ consists of user-IRS-BS reflected CFRs. The expression in (11) is simplified in (12) (at the top of next page) by separating the desired and the intrinsic interference in FBMC. Since, the FBMC pulse $p[k]$ is well FT localised, the interference caused by FT points outside the neighbourhood $\Omega_{\bar{m},\bar{n}}$ (excluding $(\bar{m},\bar{n})$) is negligible. Most of the interference comes from the first order neighborhood of $(\bar{m},\bar{n})$ which is defined as $\Omega_{\bar{m},\bar{n}} = \{(\bar{m}\pm 1,\bar{n}\pm 1),(\bar{m},\bar{n}\pm 1),(\bar{m}\pm 1,\bar{n})\}$ [11]. Since the neighbourhood $\Omega_{\bar{m},\bar{n}}$ of the FT point $(\bar{m},\bar{n})$ is very small due to well localization of the pulse $p[k]$, one can assume that the CFR is constant over the neighbourhood $\Omega_{\bar{m},\bar{n}}$. This implies that $H_{d_m} \approx H_{d_{\bar{m}}}$ and $H_{c_{i_m}} \approx H_{c_{i_{\bar{m}}}}$ for $m \in \Omega_{\bar{m},\bar{n}}$. Using the above properties, the expression in (12) can be written as,

$$y_{\bar{m},\bar{n}} = H_{\bar{m}} c_{\bar{m},\bar{n}} + \eta_{\bar{m},\bar{n}}, \quad (13)$$

where $H_{\bar{m}} = H_{d_{\bar{m}}} + \sum_{i=1}^{R} H_{c_{i_{\bar{m}}}}\phi_i$ and $c_{\bar{m},\bar{n}} = d_{\bar{m},\bar{n}} + I_{\bar{m},\bar{n}}$ is the virtual symbol at FT point $(\bar{m},\bar{n})$. The imaginary intrinsic interference is given by $I_{\bar{m},\bar{n}} = \sum_{(m,n)\in\Omega_{\bar{m},\bar{n}}} d_{m,n}\xi_{m,n}^{\bar{m},\bar{n}}$ [11]

Since $E\left[d_{m,n}(d_{m,n})^*\right] = p_m$, it follows from [11] that $E\left[c_{m,n}(c_{m,n})^*\right] \approx 2p_m$. After performing equalization in (13) and taking real part, we get

$$\Re\left(\frac{y_{\bar{m},\bar{n}}}{H_{\bar{m}}}\right) = \Re\left(d_{\bar{m},\bar{n}} + I_{\bar{m},\bar{n}}\right) + \Re\left(\frac{\eta_{\bar{m},\bar{n}}}{H_{\bar{m}}}\right)$$

Note that $\Re(\eta_{\bar{m},\bar{n}}) \sim \mathcal{CN}(0,\frac{\sigma_\eta^2}{2})$. The SNR expression for the $\bar{m}$-th sub-carrier is given by

$$\text{SNR} = \frac{|H_{\bar{m}}|^2 2p_{\bar{m}}}{\sigma_\eta^2}. \quad (14)$$

We assume a total transmission power to be P which is distributed across N sub-carriers . Let $\boldsymbol{p} = [p_1,\cdots,p_N]^T \in \mathbb{R}^{N\times 1}$, where the $\bar{m}$-th element $p_{\bar{m}} \geq 0$ denotes the amount of power allocated to the $\bar{m}$-th sub-carrier at the user. Using the SNR expression in (14), the uplink achievable spectral efficiency (SE) is obtained as,

$$r(\boldsymbol{p},\boldsymbol{\phi}) = \frac{1}{N}\sum_{\bar{m}=0}^{N-1}\log_2\left(1+\frac{|H_{\bar{m}}|^2 2p_{\bar{m}}}{\Gamma\sigma_\eta^2}\right) \quad (15)$$

where the achievable SE gap is parameterized by $\Gamma \geq 1$ due to the practical modulation and coding techniques.

III. CFR ESTIMATION FOR IRS-FBMC WAVEFORM

Considering all sub-carriers, (13) can be written as

$$\boldsymbol{y}_{\bar{n}} = \mathbf{H}\boldsymbol{c}_{\bar{n}} + \boldsymbol{\eta}_{\bar{n}}, \quad (16)$$

where $\boldsymbol{y}_{\bar{n}} = [y_{1,\bar{n}},\cdots,y_{N,\bar{n}}]^T \in \mathbb{C}^{N\times 1}$ is the $\bar{n}$-th received symbol vector across all the sub-carriers, $\boldsymbol{\eta}_{\bar{n}} = [\eta_{1,\bar{n}},\cdots,\eta_{N,\bar{n}}]^T \in \mathbb{C}^{N\times 1}$ is the corresponding noise vector such that $E\left[\boldsymbol{\eta}\boldsymbol{\eta}^H\right] = \sigma_\eta^2\mathbf{I}_N$, $\boldsymbol{c}_{\bar{n}} = [c_{1,\bar{n}},\cdots,c_{N,\bar{n}}]^T \in \mathbb{C}^{N\times 1}$ is the virtual symbol vector and $\mathbf{H}$ is $N\times N$ CFR matrix which is given by $\text{diag}(\boldsymbol{h}) = \text{diag}\left([H_1,\cdots,H_N]^T\right)$, where $\boldsymbol{h} = \boldsymbol{h}_d + \boldsymbol{h}_r = \boldsymbol{h}_d + \mathbf{H}_r\boldsymbol{\phi}$. The frame structure for the channel estimation and optimization scheme is shown in Fig.1. It consists of three phases, namely the training phase, the processing and feedback phase and the data transmission phase. The user transmits $(R+1)$ number of training symbols in the training phase. Unlike OFDM, the adjacent time domain FBMC symbols overlap with each other. To suppress this overlapping, z zeros are inserted between the adjacent training symbols, as shown in Fig.1. Thus the training phase requires $(R+1)(z+1)$ number of OQAM symbols. Later in the Section V, we show the effect of z on the MSE of the channel estimation. In practice, $z=1$ is sufficient to suppress the ISI between the adjacent training symbols. Thus, $2(R+1)$ number of OQAM symbols are required for channel estimation, which is equivalent to $(R+1)$ QAM symbols because duration of an OQAM symbol is half of the QAM symbol duration. This implies that channel estimation in both OFDM and FBMC requires the same amount of training overhead [11]. Using the training phase, the BS estimates user-BS direct channel ($\boldsymbol{h}_d$) and user-IRS-BS reflected channel ($\mathbf{H}_r$). The BS then optimizes IRS reflecting coefficients ($\boldsymbol{\phi}$) and sub-carriers transmit power allocation ($\boldsymbol{p}$) to maximize the uplink achievable SE, and then feeds back $\boldsymbol{\phi}$ to the IRS controller and $\boldsymbol{p}$ to the user in the second phase. In third phase, data transmission is carried out using the optimized values of $\boldsymbol{\phi}$ and $\boldsymbol{p}$.

Recall that the user-IRS-BS channel matrix $\mathbf{H}_r \in \mathbb{C}^{N\times R}$ consists of all reflected channels associated with the IRS elements which are superposed in the resultant user-IRS-BS composite link as $\boldsymbol{h}_r = \mathbf{H}_r\boldsymbol{\phi}$. To estimate individual channels, we use ON/OFF state control of the IRS elements. The training phase is divided into $(R+1)$ training subframes, ranging from 0 to R. Each training subframe consists of an OQAM symbol followed by z zeros, as shown in Fig. 1. We send the same training symbol in every training subframe. All the IRS elements are switched off in the zero-th training subframe. Thus, (16) can be written as

$$\boldsymbol{y}_0 = \mathbf{C}_0\boldsymbol{h}_d + \boldsymbol{\eta}_0, \quad (17)$$

where $\boldsymbol{h}_d$ is the N-point FFT of $\tilde{\boldsymbol{h}}_d$, $\mathbf{C}_0 = \text{diag}(\boldsymbol{c}_0)$ with $c_{\bar{m},0} = d_{\bar{m},0} + I_{\bar{m},0}$ being the $\bar{m}$-th element of $\boldsymbol{c}_0$. The intrinsic interference $I_{\bar{m},0}$ can be calculated as $I_{\bar{m},0} = \sum_{(m,n)\in\Omega_{\bar{m},\bar{n}}} d_{m,0}\xi_{m,n}^{\bar{m},\bar{n}}$ [11]. In the i-th training subframe, for $1 \leq i \leq R$, only the i-th IRS element is 'ON' and all the other IRS elements are switched OFF, i-e., $\boldsymbol{\phi} = \boldsymbol{e}_i$ where $\boldsymbol{e}_i$ denotes the i-th column of the identity matrix $\mathbf{I}_R$. For this case, (16) can be written as

$$\boldsymbol{y}_i = \mathbf{C}_i(\boldsymbol{h}_d + \boldsymbol{h}_{c_i}) + \boldsymbol{\eta}_i, \quad (18)$$

where $\boldsymbol{h}_{c_i}$ is the N-point FFT of $\tilde{\boldsymbol{h}}_{c_i}$. Then, based on (17) and (18), CFR of the user-IRS-BS reflected channels and

$$y_{\bar{m},\bar{n}} = \left(H_{d_{\bar{m}}} + \sum_{i=0}^{R-1} H_{c_{i_{\bar{m}}}}\phi_i\right)\left(d_{\bar{m},\bar{n}} + \underbrace{\sum_{m\neq\bar{m},n\neq\bar{n}} d_{m,n}\left(\frac{H_{d_m} + \sum_{i=0}^{R-1} H_{c_{i_m}}\phi_i}{H_{d_{\bar{m}}} + \sum_{i=0}^{R-1} H_{c_{i_{\bar{m}}}}\phi_i}\right)\xi_{m,n}^{\bar{m},\bar{n}}}_{\text{Intrinsic Interference}}\right) + \eta_{\bar{m},\bar{n}} \tag{12}$$

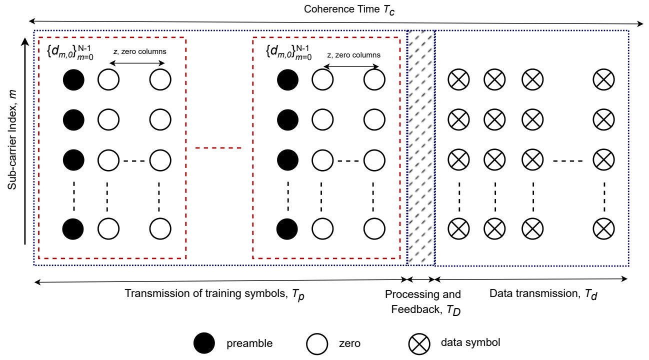


Fig. 1: Frame structure for IRS assisted FBMC waveform.

user-BS direct link channel can be estimated at the BS by using the least square (LS) estimator as follows

$$\hat{\boldsymbol{h}}_d = \mathbf{C}_0^{-1}\boldsymbol{y}_0 = \boldsymbol{h}_d + \mathbf{C}_0^{-1}\boldsymbol{\eta}_0$$
$$\hat{\boldsymbol{h}}_{c_i} = \mathbf{C}_i^{-1}\left(\boldsymbol{y_i} - \mathbf{C}_i\hat{\boldsymbol{h}}_d\right) = \boldsymbol{h}_{c_i} + \mathbf{C}_i^{-1}\boldsymbol{\eta}_i - \mathbf{C}_0^{-1}\boldsymbol{\eta}_0.$$

The overall estimated CFR is obtained as

$$\hat{\boldsymbol{h}} = \hat{\boldsymbol{h}}_d + \widehat{\mathbf{H}}_r\boldsymbol{\phi}, \tag{19}$$

where $\widehat{\mathbf{H}}_r = \left[\hat{\boldsymbol{h}}_{c_i}\cdots\hat{\boldsymbol{h}}_{c_R}\right]^T \in \mathbb{C}^{N\times R}$. We next derive the upper bound on CFR estimation MSE. The average mean-square-error (MSE) of the estimated CFR is derived as

$$\begin{aligned}\varepsilon &= \mathbb{E}\left\{\left\|\hat{\boldsymbol{h}}_d + \widehat{\mathbf{H}}_r\boldsymbol{\phi} - \boldsymbol{h}_d - \mathbf{H}_r\boldsymbol{\phi}\right\|^2\right\}\\ &= \mathbb{E}\left\{\left\|\left(\hat{\boldsymbol{h}}_d - \boldsymbol{h}_d\right) + \sum_{i=1}^{R}\left(\hat{\boldsymbol{h}}_{c_i} - \boldsymbol{h}_{c_i}\right)\phi_i\right\|^2\right\}\\ &\leq \mathbb{E}\left\{\left\|\mathbf{C}_0^{-1}\boldsymbol{\eta}_0\right\|^2\right\} + \sum_{i=1}^{R}\mathbb{E}\left\{\left\|\left(\mathbf{C}_i^{-1}\boldsymbol{\eta}_i - \mathbf{C}_0^{-1}\boldsymbol{\eta}_0\right)\right\|^2\right\}\\ &\leq (R+1)\,\mathbb{E}\left\{\left\|\mathbf{C}_0^{-1}\boldsymbol{\eta}_0\right\|^2\right\} + \sum_{i=1}^{R}\mathbb{E}\left\{\left\|\left(\mathbf{C}_i^{-1}\boldsymbol{\eta}_i\right)\right\|^2\right\}\\ &\leq \frac{(2R+1)\,N\sigma_\eta^2}{2p_m}.\end{aligned} \tag{20}$$

IV. Joint Optimization of Reflecting Coefficients and Sub-carrier Power

A. Problem Formulation

The BS optimizes the IRS reflecting coefficients $\boldsymbol{\phi}$ and sub-carriers transmit power allocation $\boldsymbol{p}$ in the second phase using the overall estimated CFR $\hat{\boldsymbol{h}}$ to maximize the uplink achievable SE. The SE with the estimated CFR is expressed as

$$r\left(\boldsymbol{p},\boldsymbol{\phi}|\hat{\boldsymbol{h}}_d,\widehat{\mathbf{H}}_r\right) = \left(1 - \frac{T_p + \tau_D}{T_c}\right)\frac{1}{N}\times \sum_{\bar{m}=0}^{N-1}\log_2\left(1 + \frac{\left|\hat{H}_{\bar{m}}\right|^2 2p_{\bar{m}}}{\Gamma\sigma_\eta^2}\right) \tag{22}$$

where T_c, T_p and τ_D denote the coherence time, time taken for CFR estimation and the delay for processing and feedback, respectively. Here, we develop a joint optimization framework to obtain the optimal IRS reflecting coefficients and sub-carriers transmit power allocation with an objective to maximize the uplink achievable SE given in (22), considering the CFR estimates. To this end, the optimization problem is given as

$$\begin{aligned}\text{(P1)} \quad :\underset{\boldsymbol{p},\boldsymbol{\phi}}{\text{maximize}} \quad & \sum_{\bar{m}-0}^{N-1}\log_2\left(1 + \frac{\left|\hat{H}_{\bar{m}}\right|^2 2p_{\bar{m}}}{\Gamma\sigma_\eta^2}\right)\\ \text{subject to} \quad & \sum_{\bar{m}=0}^{N-1} p_{\bar{m}} \leq P, & (23)\\ & p_{\bar{m}} \geq 0,\ \forall\bar{m}\in\left\{1 \quad \cdots \quad N\right\} & (24)\\ & |\phi_i| \leq 1,\ \forall i\in\left\{1 \quad \cdots \quad R\right\}, & (25)\end{aligned}$$

where $\hat{H}_{\bar{m}} = \hat{H}_{d_{\bar{m}}} + \sum_{i=1}^{R}\hat{H}_{c_{i_{\bar{m}}}}\phi_i$. The objective function of the above problem (P1) is non-concave over $\boldsymbol{\phi}$, which makes this optimization problem to be non-convex in nature. Considering the above issues and the fact that the objective function consists of both the variables $\boldsymbol{p}$ and $\boldsymbol{\phi}$, we obtain a sub-optimal solution to (P1) by iteratively optimizing $\boldsymbol{p}$ and $\boldsymbol{\phi}$, keeping one variable to be fixed at each iteration.

B. Optimization of Sub-carriers Transmit Power Allocation

Here, we use water filling (WF) algorithm [13] to obtain optimal sub-carriers transmit power $\boldsymbol{p}$ using the knowledge of the IRS reflecting coefficients $\boldsymbol{\phi}$ and the estimated CFR $\hat{\boldsymbol{h}}$ given in (19). To this end, the optimal power allocated to the $\bar{m}$-th sub-carrier is obtained as

$$p_{\bar{m}} = \left(\frac{1}{v_u} - \frac{1}{v_{\bar{m}}}\right)^+, \forall\bar{m}\in\left\{1 \quad \cdots \quad N\right\} \tag{26}$$

where $(x)^+ \triangleq \max(0,x)$, $v_{\bar{m}} = \left|\hat{H}_{\bar{m}}\right|^2 / \left(\Gamma\sigma_\eta^2\right)$ is effective channel-to-noise power ratio (CNR) for the $\bar{m}$-th sub-carrier. The cut-off CNR v_u satisfies,

$$\sum_{\bar{m}-0}^{N-1}\left(\frac{1}{v_u} - \frac{1}{v_{\bar{m}}}\right)^+ = P. \tag{27}$$

$$\tilde{f}_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right) \geq \tilde{a}_{\bar{m}}^2 + \tilde{b}_{\bar{m}}^2 + 2\tilde{a}_{\bar{m}}^2\left(a_{\bar{m}} - \tilde{a}_{\bar{m}}\right) + 2\tilde{b}_{\bar{m}}^2\left(b_{\bar{m}} - \tilde{b}_{\bar{m}}\right) \triangleq f_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right) \tag{21}$$

C. Optimization of IRS Reflecting Coefficients

We next develop an optimization framework to obtain optimal IRS reflecting coefficients $\boldsymbol{\phi}$, considering the knowledge of power allocation $\boldsymbol{p}$ and estimated CFR $\hat{\boldsymbol{h}}$. Based on the knowledge of $\boldsymbol{p}$, we can simplify the Problem (P1) as

$$\text{(P1.1)} \ : \underset{\boldsymbol{p},\boldsymbol{\phi}}{\text{maximize}} \quad \sum_{\bar{m}-0}^{N-1} \log_2\left(1 + \frac{\left|\hat{H}_{\bar{m}}\right|^2 2p_{\bar{m}}}{\Gamma\sigma_\eta^2}\right)$$
$$\text{s. t.} \quad |\phi_i| \leq 1, \ \forall i \in \left\{1 \ \ \cdots \ \ R\right\} \tag{28}$$

(P1.1) is a non-convex optimization problem, therefore, to get an approximate solution of the above problem we use the successive convex approximation (SCA) technique [14]. According to this technique, we first frame the following optimization problem by considering a set of auxiliary variables $t_{\bar{m}}$'s, $a_{\bar{m}}$'s, and $b_{\bar{m}}$'s

$$\text{(P1.1')}: \underset{\boldsymbol{\phi},\{t_{\bar{m}}\},\{a_{\bar{m}}\},\{b_{\bar{m}}\}}{\text{maximize}} \quad \sum_{\bar{m}-0}^{N-1} \log_2\left(1 + \frac{t_{\bar{m}} 2p_{\bar{m}}}{\Gamma\sigma_\eta^2}\right)$$
$$\text{subject to} \quad |\phi_i| \leq 1, \ \forall i \in \left\{1 \ \ \cdots \ \ R\right\} \tag{29}$$
$$a_{\bar{m}} = \Re\left\{\hat{\boldsymbol{h}}_d + \widehat{\mathbf{H}}_r\boldsymbol{\phi}\right\}, \ \forall \bar{m} \in \mathcal{N} \tag{30}$$
$$b_{\bar{m}} = \Im\left\{\hat{\boldsymbol{h}}_d + \widehat{\mathbf{H}}_r\boldsymbol{\phi}\right\}, \ \forall \bar{m} \in \mathcal{N} \tag{31}$$
$$t_{\bar{m}} \leq \tilde{f}_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right), \ \forall \bar{m} \in \mathcal{N} \tag{32}$$

Here the set $\mathcal{N}$ is defined as $\mathcal{N} = \{1, \cdots, N\}$ and $\tilde{f}_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right) \triangleq a_{\bar{m}}^2 + b_{\bar{m}}^2$ which is a non-convex constraint. We thus reconstruct (P1.1') as the following optimization problem, by converting (32) to an affine constraint

$$\text{(P1.2)} \ : \underset{\boldsymbol{\phi},\{t_{\bar{m}}\},\{a_{\bar{m}}\},\{b_{\bar{m}}\}}{\text{maximize}} \quad \sum_{\bar{m}-0}^{N-1} \log_2\left(1 + \frac{t_{\bar{m}} 2p_{\bar{m}}}{\Gamma\sigma_\eta^2}\right)$$
$$\text{s. t.} \quad (29), (30), (31) \tag{33}$$
$$t_{\bar{m}} \leq f_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right) \ \forall \bar{m} \in \mathcal{N}, \tag{34}$$

where $f_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right)$ is given in (21) (top of this page) is an affine function over $a_{\bar{m}}$ and $b_{\bar{m}}$, and obtained as the first order approximation of $\tilde{f}_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right)$. It is also observed that $f_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right)$ acts as a lower bound of $\tilde{f}_{\bar{m}}\left(a_{\bar{m}}, b_{\bar{m}}\right)$ for any $\tilde{a}_{\bar{m}}$ and $\tilde{b}_{\bar{m}}$. We now observe that (P1.2) is a convex optimization problem which thus can be solved using CVX software [15]. The resulting optimal values of $a_{\bar{m}}$ and $b_{\bar{m}}$ used to obtain the approximate solution to (P1.1') by updating the variables $\tilde{a}_{\bar{m}}$ and $\tilde{b}_{\bar{m}}$ successively for $\bar{m} \in \mathcal{N}$, as describe in Algorithm 1. Next, using the updated variables $\{\tilde{a}_{\bar{m}}\}$ and $\{\tilde{b}_{\bar{m}}\}$ in an iterative manner, we obtain the optimal IRS reflecting coefficients which satisfy the convergence condition mentioned in Algorithm 1. As a result, we achieve the approximate solution to (P1.1) for the given power allocation $\boldsymbol{p}$.

Algorithm 1: Approximate solution to the Problem P1.1 for given Power Allocation

Input: $\hat{\boldsymbol{h}}_d$, $\hat{\mathbf{H}}_r$, $\boldsymbol{p}$, Γ, σ_η^2, N, R, $\tilde{\boldsymbol{\phi}}$
Output: $\boldsymbol{\phi}$
Initialization: $\{\tilde{a}_{\bar{m}}\} = \Re\left\{\hat{\boldsymbol{h}}_d + \hat{\mathbf{H}}_r\boldsymbol{\phi}\right\}$, $\left\{\tilde{b}_{\bar{m}}\right\} = \Im\left\{\hat{\boldsymbol{h}}_d + \hat{\mathbf{H}}_r\boldsymbol{\phi}\right\}$, $\forall \ \bar{m} \in \{1 \cdots N\}$.
repeat
  Using $\{\tilde{a}_{\bar{m}}\}$, $\left\{\tilde{b}_{\bar{m}}\right\}$ and $\boldsymbol{p}$, solve (P1.2) through CVX to get the optimal values of $\{a_{\bar{m}}\}$, $\{b_{\bar{m}}\}$ and $\boldsymbol{\phi}$.
  $\{\tilde{a}_{\bar{m}}\} = \{a_{\bar{m}}\}$, $\left\{\tilde{b}_{\bar{m}}\right\} = \{b_{\bar{m}}\}$, $\forall \ \bar{m} \in \{1 \cdots N\}$
until *the objective function of (P1.1) converges for the achieved* $\boldsymbol{\phi}$
return: $\boldsymbol{\phi}$

Algorithm 2: Alternative Optimization for solving (P1)

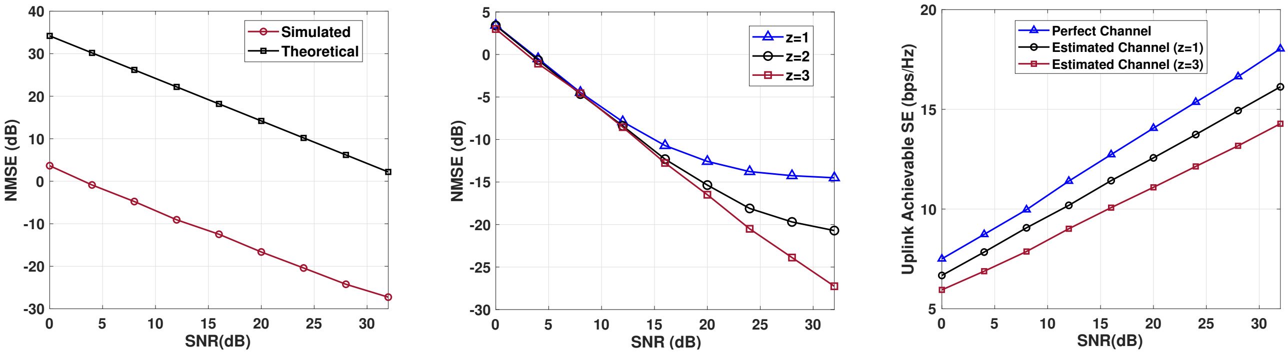
Input: $\hat{\boldsymbol{h}}_d$, $\hat{\mathbf{H}}_r$, P, Γ, σ_η^2, N, R, $\boldsymbol{\phi}_0$
Output: $\boldsymbol{p}^\star$, $\boldsymbol{\phi}^\star$.
Initialization: $\boldsymbol{\phi} = \boldsymbol{\phi}_0$.
repeat
  Find the optimal power allocation $\boldsymbol{p}$ using Water filling (WF) algorithm as per (26) and (27) for the fixed values of IRS reflecting coefficients $\boldsymbol{\phi}$.
  Use Algorithm 1 to update $\boldsymbol{\phi}$ by setting $\tilde{\boldsymbol{\phi}} = \boldsymbol{\phi}$, for the fixed values of optimal power allocation $\boldsymbol{p}$.
until *the objective function of (P1) converges for the acquired* $\boldsymbol{p}$ *and* $\boldsymbol{\phi}$
return: $\boldsymbol{p}^\star = \boldsymbol{p}$, $\boldsymbol{\phi}^\star = \boldsymbol{\phi}$.

Considering the optimal reflecting coefficients obtained using Algorithm 1, we next develop a framework given in Algorithm 2 to solve the Problem (P1) as follows. Considering some random IRS reflecting coefficients $\boldsymbol{\phi}_0$ as the initial value of $\boldsymbol{\phi}$, we first obtain the power allocation $\boldsymbol{p}$ which further used to update the coefficients $\boldsymbol{\phi}$ via Algorithm 1. These steps are successively executed until the convergence is reached. At the end, we obtain the optimal IRS reflecting coefficients and sub-carriers transmit power allocation. Here, we consider the initial IRS reflecting coefficients by setting the amplitude $\beta_i = 1$ and the phase θ_i to be uniformly distributed over the interval $[0, 2\pi)$.

V. Simulation Results

We consider FBMC/OQAM system with $N = 128$ sub-carriers. The wireless channel for user-BS direct link, IRS-BS link and user-IRS link is assumed to be Rayleigh faded having maximum delay spread of $L_0 = 6$, $L_1 = 2$ and $L_2 = 5$, respectively. Each channel coefficient follows circularly-symmetric complex Gaussian distribution. We consider $R = 20$ IRS elements for all the simulations.

In Fig. 2(a), we show the comparison between the simulated and theoretical values of normalized mean-squared-error (NMSE) versus SNR for the overall CFR estimates $\hat{\mathbf{h}}$, with $z = 3$ (three zeros are inserted between the adjacent training symbols). We observe that the theoretical NMSE act as an upper bound for the simulated one, as also emphasized in (20). Figure 2(b) shows the performance of simulated NMSE for $z = \{1, 2, 3\}$. It can be observed from this



(a) (b) (c)

Fig. 2: a) Comparison of the simulated and theoretical values of NMSE. (b) Effect of z on NMSE. (c) Effect of channel estimation and z on uplink achievable SE.

figure that the NMSE for $z = 3$ is less than that of $z = 2$ which is less than that of $z = 1$ for high SNR regime. This happens due to the fact that the insertion of less number of zeros between the adjacent training symbols increases the interference which in turn saturates the NMSE at the high SNR values.

For this study, we consider SNR gap of $\Gamma = 8.8$ dB and coherence time of $T_c = 200$ FBMC/OQAM symbols. Figure 2 (c) compares the achievable SE using the proposed joint optimization scheme (Algorithm 2) in the presence of the estimated and perfect CFRs. It also demonstrates the effect of guard symbols (zeros) on the achievable SE. It can be observed that the achievable SE with the estimated CFR is less than that of assuming perfect CFR, which is due to the imperfect estimate of CFR. It can also be observed that uplink achievable SE for $z = 3$ is less than that of $z = 1$ because of loss of spectral efficiency due to insertion of zeros between two adjacent training symbols. We also see that the proposed scheme performs close to the perfect CFR case with $z = 1$.

VI. Conclusions

We designed an IRS assisted FBMC waveform for future wireless communication systems. Utilizing a novel frame structure, CFR estimation scheme is investigated by selecting IRS reflecting elements using ON/OFF method. We also proposed successive convex approximation based optimization technique to obtain optimal values of IRS reflecting coefficients and sub-carriers transmit power. Our simulation results showed that the achievable SE by using the proposed channel estimation and optimization technique is close to the one achieved by assuming perfect knowledge of CFR.

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